

25th Inverse Days

University of Jyväskylä, Finland

Conference Centre Paviljonki

16–18 December, 2019

Conference information

25th Inverse Days

The "Inverse Days" conference is the annual meeting of the Finnish Inverse Problems Society (FIPS), and is part of the activity of the Finnish Centre of Excellence in Inverse Problems Research. This 25th edition is jointly organized by the University of Jyväskylä and the FIPS. You are warmly welcome in Jyväskylä!

Local organizing committee (University of Jyväskylä):

- Joonas Ilmavirta (Chair)
- Giovanni Covi
- Miia Honkonen
- Shiqi Ma
- Keijo Mönkkönen
- Leyter Potenciano-Machado
- Jesse Railo
- Mikko Salo

Conference webpage:

<https://www.jyu.fi/science/en/math/research/inverse-problems/id2019/>

<http://r.jyu.fi/yVK>



Figure 1: Scan QR code, go to the webpage

1 PROGRAM

Day 1, Monday 16

10:00 – 10:45 *Registration (1st floor lobby)*

10:45 – 11:05 *Conference opening (Wivi)*

	Wivi
11:05	Samuli Siltanen
11:20	Special talk: <i>25 years of Inverse Days</i>
11:20	Maarten V. de Hoop
12:00	Keynote talk: <i>Spectral inverse problems for the earth</i>

12:00 – 13:00 *Lunch (restaurant)*

12:00 – 13:30 *Board meeting & poster preparation (cabinet & 2nd floor lobby)*

	Wivi	Alvar
13:30	Teemu Tyni	Remo Kretschmann
13:55	<i>Numerical results on Saito's uniqueness theorem for inverse scattering</i>	<i>Generalised modes in Bayesian inverse problems</i>
13:55	Meghdoot Mozumder	Tapio Helin
14:20	<i>Truncated Fourier series based time-domain diffuse optical tomography</i>	<i>Hyperparameter estimation in Bayesian inverse problems: consistency of MAPs</i>
14:20	Mikael Laaksonen	Jonatan Lehtonen
14:45	<i>Deterministic approximation of inverse boundary spectral problems via stochastic collocation</i>	<i>Estimating hyperparameters of a hierarchical model using Bayesian filtering</i>

14:45 – 15:15 *Coffee (restaurant)*

	Wivi
15:15	Tatiana Bubba
15:40	Plenary talk: <i>Learning the Invisible: Limited Angle Tomography, Shearlets and Deep Learning</i>
15:40	Sara Sommariva
16:05	Plenary talk: <i>Optimal regularization technique for the estimation of the cross-power spectrum in underdetermined, dynamical inverse problems.</i>
16:10	Poster authors
17:00	Short talks

17:00 – 17:30 *Poster session (2nd floor lobby)*

17:30 – 19:00 *Icebreaker (2nd floor lobby)*

Day 2, Tuesday 17

8:45 – 9:00 *Registration (1st floor lobby)*

	Wivi	Alvar
9:00	Luca Ratti	Emilia Blåsten
9:25	<i>A Convolutional Neural Networks approach for sparsity-promoting regularization in CT: theoretical results</i>	<i>Detecting blockages in water supply networks using boundary control</i>
9:25	Mathilde Galinier	Leyter Potenciano-Machado
9:50	<i>Unrolled ISTA and convolutional neural networks for limited-angle tomography reconstruction</i>	<i>A resolvent estimate for the magnetic Schrödinger operator in the presence of short and long-range potentials</i>
9:50	Jalo Nousiainen	Shiqi Ma
10:15	<i>Inverse problems and deep learning in predictive control for adaptive optics</i>	<i>Determining a random Schrödinger equation</i>

10:15 – 10:45 *Coffee (restaurant)*

	Wivi
10:45	Marco Mazzucchelli
11:10	<i>Spectral characterizations of Besse and Zoll Reeb flows</i>
11:10	Tuomo Valkonen
11:35	<i>First-order primal-dual methods for non-linear inverse problems</i>
11:35	Yavar Kian
12:00	<i>Inverse problem for diffusion equations</i>

12:00 – 13:00 *Lunch (restaurant)*

12:00 – 13:30 *Women in Inverse Problems (cabinet)*

	Wivi	Alvar
13:30	Lauri Ylinen	Otto Lamminpää
13:55	<i>Analysis of a Dynamical System Modelling Lasers and Applications for Optical Neural Networks</i>	<i>UQ for satellite measurements of CO₂ using MCMC</i>
13:55	Rahul Yadav	Vesa Kaarnioja
14:20	<i>Deep convolutional neural networks for estimating porous material properties using microwave tomography</i>	<i>Uncertainty quantification for PDE-constrained optimal control problems using quasi-Monte Carlo methods</i>
14:20	Andreas Hauptmann	Bjørn Christian Skov Jensen
14:45	<i>Learned image reconstruction for large scale tomographic imaging</i>	<i>Wave speed uncertainty in Acousto-Electric Tomography</i>

14:45 – 15:15 *Coffee (restaurant)*

	Wivi
15:15	François Monard
15:40	<i>The geodesic X-ray transform on disks of constant curvature</i>
15:40	Rashmi Murthy
16:05	<i>Classification of the stroke using Neural Networks in Electrical Impedance Tomography</i>

16:10 – 17:30 *FIPS meeting*

18:30 *Transportation to dinner (in front of Paviljonki)*

19:00 *Dinner (Scandic Hotel Laajavuori)*

22:00 *Transportation from dinner (in front of dinner venue)*

Day 3, Wednesday 18

	Wivi	Alvar
9:00	Samuli Siltanen	Giovanni Covi
9:25	<i>Gray-box machine learning for electrical impedance tomography</i>	<i>An inverse problem for the fractional Schrödinger equation in a magnetic field</i>
9:25	Ville Kolehmainen	Nataliia Kinash
9:50	<i>Non-linear difference reconstruction method for monitoring of cerebral haemorrhage</i>	<i>An inverse problem for a generalized fractional derivative with an application in reconstruction of time- and space-dependent sources in fractional diffusion and wave equations</i>
9:50	Aynur Cöl	Fredrik Hildrum
10:15	<i>Deep Learning based Virtual Hybrid Edge Detection Method in Electrical Impedance Tomography for Stroke Classification</i>	<i>Nonlinear Lavrentiev regularization of monotone ill-posed equations</i>

10:15 – 10:45 *Coffee (restaurant)*

	Wivi	Alvar
10:45	Valentina Candiani	Petri Kuusela
11:10	<i>Monotonicity-based reconstruction of extreme inclusions in electrical impedance tomography</i>	<i>Imaging of moisture flows in cement-based materials using X-ray computer tomography and electrical capacitance tomography</i>
11:10	Guanghui Liang	Teemu Sahlström
11:35	<i>Shape-based image reconstruction of electrical/ultrasonic dual-modality tomography</i>	<i>Modelling of errors due to uncertainties in ultrasound sensor locations in photoacoustic tomography</i>
11:35	Henrik Garde	Rasmus Backholm
12:00	<i>Optimal depth-dependent distinguishability bounds for EIT</i>	<i>Simultaneous Reconstruction of Emission and Attenuation in Passive Gamma Emission Tomography of Spent Nuclear Fuel</i>

12:00 – 13:15 *Lunch (restaurant)*

	Wivi	Alvar
13:15	Matteo Santacesaria	Teemu Saksala
13:40	<i>Infinite-dimensional inverse problems with a finite number of measurements</i>	<i>Generic uniqueness and stability for mixed ray transform</i>
13:40	Topi Kuutela	Tommi Brander
14:05	<i>Developments on the logarithmic forward map of electrical impedance tomography</i>	<i>1D source detection with multiple frequencies</i>

14:05 – 14:35 *Coffee (restaurant)*

	Wivi
14:35	Lauri Oksanen
15:00	<i>The light ray transform in stationary and static Lorentzian geometries</i>
15:00	Matti Lassas
15:25	<i>New deep neural networks solving non-linear inverse problems</i>

15:25 – 15:35 *Ending*

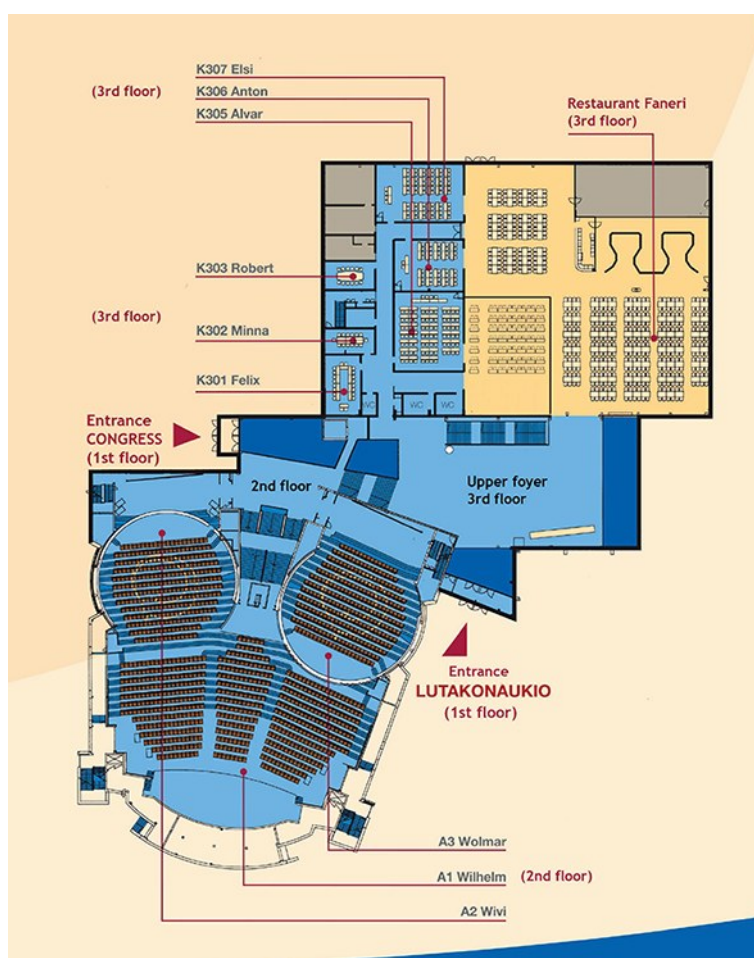
2 PRACTICAL INFORMATION

2.1 Registration

A registration desk can be found at the conference venue on the first floor lobby. Registration begins at 10:00 on Monday, and the opening is scheduled at 10:45 in the Wivi hall. If you have any questions during the conference, please contact the registration desk or any of the local organizers.

2.2 Venue

The venue of the event is the Paviljonki Conference Centre, in the immediate vicinities of the station. The scientific program takes place in the Wivi and Alvar halls. The locations of the lecture halls are shown in the following map, as well as those of the other main points of interest inside the conference centre:



2.3 Internet

Internet is provided by the Paviljonki Conference Centre. For the required network name, username and password, please refer to the ticket included in your conference material. It is also possible at all times to ask the organizers or the personal of the conference centre for help.

2.4 Lunch and coffee

Lunch is available at the second floor restaurant inside the conference centre. The coffee breaks (including snacks) are also served at the same restaurant.

2.5 Icebreaker

The customary icebreaker takes place on the second floor lobby. It is free of charge to all registered participants, and includes both drinks and light dining options.

2.6 Dinner

The conference dinner will be served at Scandic Hotel Laajavuori, close to the renowned ski resort. Transportation to and from the venue of the event has been organized as described in the Program. The dinner and transportation are free of charge to all registered participants.

2.7 Posters

The posters will be introduced on Monday with short talks and a poster session. Afterward, they will be left exposed for the whole duration of the conference. The abstracts of the posters can be found in Section 4 of the present document. Those presenting a poster are requested to bring it by 13:00 on Monday for the necessary preparations.

2.8 Local transportation

The conference venue is located in the immediate vicinities of the city centre and train station. Taxis are available at the taxi rank at the train station, or can be reserved by calling the phone number +358 100 6900. The organizers and the staff of Paviljonki would also be happy to arrange a taxi for you. Please be aware that credit card payments and high denomination banknotes are not accepted on local buses.

2.9 Local activities

It is possible to visit the sites <https://jcb.fi/en> and <https://visitjyvaskyla.fi/en> for non-scientific activities in Jyväskylä. The local organizers will also be happy to provide further information.

2.10 Feedback

Your opinion is very valuable for us. Please help us improve by filling the feedback form at <http://r.jyu.fi/yVK>

We are grateful for your time!

3 TALK ABSTRACTS

3.1 Day 1, Monday 16

3.1.1 Wivi 11:20 – 12:00

Spectral inverse problems for terrestrial planets

MAARTEN V. DE HOOP (Rice University)

We introduce an appropriate form and present an analysis of the elastic-gravitational system of equations describing the free oscillations of a rotating terrestrial planet. We highlight the complications associated with the presence of a fluid outer core and essential spectrum. We present the analysis of an inverse problem for a spherically symmetric model (such as PREM) and give, via a trace formula, a spectral rigidity result. We then focus on phase boundaries (the free surface and core-mantle boundary) and the associated semiclassical analysis of modes guided along these, while pinching the planet. In particular, we present Bohr–Sommerfeld rules for Love and Rayleigh modes and the recovery of a Lamé parameter from the associated semiclassical spectra “near” the mentioned boundaries. We conclude with a brief outlook on inverse problems with Love and Rayleigh resonances.

Joint research with S. Holman, A. Iantchenko, J. Ilmavirta, S. Jimbo, G. Nakamura, H. Pham, J. Shi and J. Zhai.

3.1.2 Wivi 13:30 – 14:45

Numerical results on Saito’s uniqueness theorem for inverse scattering

TEEMU TYNI (University of Helsinki)

We consider an inverse scattering problem for a Schrödinger operator $-\Delta + V$ in two dimensions. In scattering problems in the frequency domain one considers the equation $-\Delta u + Vu = k^2 u$, where k is the wavenumber. The scattering solution to this problem is the sum of an incident field and a scattered field. In practice one probes the unknown potential V by sending in an incident field and then measuring the resulting scattered field far away from the potential V . This results in so-called scattering amplitude. In 1982 Y. Saitō proved that if we are given the scattering amplitude for all incident angles and all measurement angles for all high wavenumbers then this data corresponds to a unique potential function V . In this talk we discuss the numerical implementation of Saitō’s formula and show how it can be used to recover the potential V from the scattering data.

Truncated Fourier series based time-domain diffuse optical tomography

MEGHDOOT MOZUMDER (University of Eastern Finland)

Time-domain diffuse optical tomography (TD-DOT) uses pulsed infrared lasers for imaging spatially varying optical parameters of biological tissues. The TD-DOT image reconstruction problem has been solved using several approaches. These include using the whole time-resolved measurement data and using integral-transform based moments of the time resolved data (e.g, Mellin and Laplace transforms). Use of such moments has led to a significant reduction of computation time due to compression of TD-DOT measurement data. Nevertheless, using only one moment is inadequate to reconstruct both absorption and scattering parameters simultaneously. In this work, we propose utilizing a truncated Fourier series approximation for TD-DOT. The approach was evaluated using numerical simulations. Using this approximation, we obtained optical parameter estimates with good contrast and minimal parameter cross-talk. The estimates showed accuracy comparable to using the whole time-resolved data, and used low computational time and resources. This is a joint work with Tanja Tarvainen.

Deterministic approximation of inverse boundary spectral problems via stochastic collocation

MIKAEL LAAKSONEN (Lappeenranta-Lahti University of Technology)

Considerable effort has been devoted to the development of efficient numerical methods for quantifying the effects of uncertain input variables in physical systems. In the context of Bayesian inverse problems, these methods can naturally be applied to compute an approximation of the associated forward map, from which statistics of the posterior distribution may then be derived. In very recent years, various methods have also been proposed for solving eigenvalue problems with uncertain inputs. Therefore, a natural next step is to apply these methods for the numerical approximation of inverse boundary spectral problems. In this talk we consider the reconstruction of a one-dimensional Schrödinger operator from spectral data. More precisely, we aim to reconstruct the potential function from noisy measurements of the operator's Dirichlet and Dirichlet-Neumann eigenvalues. We assume that this potential function admits a parametrization with respect to a countable number of independent random variables. A polynomial approximation of the associated forward map, i.e., the map from the parametrized potential to the eigenvalues, is resolved using a stochastic collocation method defined on a sparse grid. Statistics of the posterior distribution can then easily be computed from this polynomial approximation.

3.1.3 Alvar 13:30 – 14:45

Generalised modes in Bayesian inverse problems

REMO KRETSCHMANN (School of Engineering Science, Lappeenranta-Lahti University of Technology)

We examined generalised modes in nonparametric Bayesian inverse problems. We gave examples for measures where the common definition of a (strong) mode excludes points that we would intuitively consider a mode, whereas the generalised definition covers these points. We showed that under certain conditions, which are fulfilled for a number of widely used measures, strong and generalised modes coincide. Then we considered inverse problems with a uniform prior that has compact support, characterised the generalised posterior modes as minimisers of a canonical objective functional and showed consistency of the generalised MAP estimator in the presence of Gaussian noise.

Hyperparameter estimation in Bayesian inverse problems: consistency of MAPs

TAPIO HELIN (Lappeenranta-Lahti University of Technology)

Often in Bayesian inverse problems it is computationally infeasible to sample the full posterior, by for example using MCMC methods, and so instead maximum a posteriori (MAP) estimates are found. Whilst these are relatively cheap to compute, a key drawback is their lack of parametrization invariance. This is an especially significant issue when hierarchical priors are employed, particularly in the non-parametric setting. In this talk we discuss the effect of the choice of parameterization when the prior distribution is conditionally Gaussian. Specifically we consider the centred parametrization, the natural parametrization in which the unknown state is solved for directly, and the non-centred parameterization, which arises by considering dimension-robust sampling algorithms.

Estimating hyperparameters of a hierarchical model using Bayesian filtering

JONATAN LEHTONEN (University of Helsinki)

In Bayesian hierarchical modeling, the prior distribution is assumed to depend on some parameters. This talk focuses on the estimation of these so-called hyperparameters, particularly in cases with a high-dimensional forward model and only a few hyperparameters. I discuss the estimation of these parameters by Bayesian filtering, and the challenges that arise due to the dimensionality of the forward model. Finally, I present an example related to turbulence profiling, which provides necessary prior information for correcting the effect of turbulence on next-generation ground-based telescopes. Bayesian filtering allows tracking this turbulence profile in real time, improving the temporal resolution of turbulence profiling from minutes to less than a second, which also has the potential to reveal new information about turbulence dynamics.

Learning the Invisible: Limited Angle Tomography, Shearlets and Deep Learning

TATIANA BUBBA (University of Helsinki)

Limited angle geometry is still a rather challenging modality in computed tomography (CT), in which entire boundary sections are not captured in the measurements making the reconstruction a severely ill-posed inverse problem. Compared to the standard filtered back-projection, iterative regularization-based methods help in removing artifacts but still cannot deliver satisfactory reconstructions. Based on the result that limited tomographic data sets reveal parts of the wavefront (WF) set in a stable way and artifacts from limited angle CT have directional properties, we propose a hybrid reconstruction framework that combines model-based sparse regularization with data-driven deep learning. The core idea is to solve the compressed sensing formulation associated to the limited angle CT problem to recover the so called “visible” part of WF and learning via a convolutional neural network architecture the “invisible” ones, which provably cannot be handled by model-based methods. Such a decomposition into visible and invisible parts is achieved using the shearlet transform that allows to resolve WF sets in the phase space. Our numerical experiments show that our approach surpasses both pure model- and more data-based reconstruction methods, while offering an (heuristic) understanding of why the method works, providing a more reliable approach especially for medical applications. This is a joint work with G. Kutyniok, M. Lassas, M. März, W. Samek, S. Siltanen and V. Srinivasan.

Optimal regularization technique for the estimation of the cross-power spectrum in underdetermined, dynamical inverse problems

SARA SOMMARIVA (Department of Mathematics, University of Genova, Italy)

We consider the problem of estimating the statistical dependence between the components of a multivariate stochastic process when only indirect measurements are available. This problem raises in many application as in the study of functional connectivity from magnetoencephalographic (MEG) data i.e. in the study of the statistical interaction between different brain areas from the electromagnetic field they generate outside the scalp. From a mathematical point of view, the study of functional connectivity involves at first the solution of an ill-posed, dynamic, linear inverse problem $\mathbf{y}(\tau) = \mathbf{L}\mathbf{x}(\tau) + \mathbf{e}(\tau)$, $\tau = 1, \dots, t$, where $\mathbf{y}(\tau) \in \mathbb{R}^m$ is the data recorded by m sensors at time τ , $\mathbf{L} \in \mathbb{R}^{m \times n}$ is the forward operator, $\mathbf{x}(\tau) \in \mathbb{R}^n$ is the unknown activity in n points of a brain discretization \mathcal{V} , and $\mathbf{e}(\tau)$ is noise. Eventually, the statistical dependence between the components of $\mathbf{x}(\tau)$ is quantified through suitable connectivity metrics, often computed in the frequency domain from the cross-power spectrum $S^{\mathbf{x}}(f)$ of the stochastic process $\{\mathbf{x}(\tau)\}_{\tau=1}^t$. Two main issues affect this approach. Problem I: the MEG inverse problem is highly undetermined as the brain discretization \mathcal{V} typically comprises $n \sim 10000$ candidate source locations while typical MEG helmets include $m \sim 300$ sensors. Problem II: the MEG inverse problem is often solved by means of Tikhonov regularization, where a key step consists in choosing the regularization parameter. However, in the connectivity framework, an unexpected parameter tuning issue may rise, as the value of the regularization parameter that provides the best estimate of the unknown $\mathbf{x}(\tau)$ may be suboptimal for the subsequent computation of the connectivity metrics [1]. Here we investigate these two issues. More specifically, we present a model-reduction technique for underdetermined linear inverse problems based on a recent unsupervised clustering algorithm called FLAME, fuzzy clustering by local approximation of membership [2]. As far as Problem II is concerned, we proceed with analytical calculations assuming both the noise $\mathbf{e}(\tau)$ and the unknown $\mathbf{x}(\tau)$ to be realizations of white noise Gaussian processes and we prove that the value of the regularization parameter that minimizes the ℓ_2 -error in estimating $S^{\mathbf{x}}(f)$ is less than half of the value of the regularization parameter that minimizes the ℓ_2 -error in estimating $\mathbf{x}(\tau)$ [3].

[1] A. Hincapié et al. *Comput Intel Neurosc*, 2006

[2] S. Sommariva, N. Puthanmadam Subramaniam, and L. Parkkonen. In preparation.

[3] E. Vallarino, S. Sommariva, P. Michele, and A. Sorrentino. preprint at <https://arxiv.org/abs/1909.01266>

3.2 Day 2, Tuesday 17

3.2.1 Wivi 9:00 – 10:15

A Convolutional Neural Networks approach for sparsity-promoting regularization in CT: theoretical results

LUCA RATTI (University of Helsinki)

The Iterative Soft Thresholding is one of the most common reconstruction algorithm employed in sparsity-promoting regularization of inverse problems. It is possible to show that, under rather general assumptions on the forward operator, the unfolded iterations of ISTA can be interpreted as the successive layers of a Convolutional Neural Network. In this talk, I will present how this idea allows not only to design a Neural Network architecture which is able to outperform the effectiveness of ISTA, but also to derive some rigorous convergence estimates of reconstructed solution. Preliminary results have been obtained in the context of limited-angle CT, enforcing sparsity with respect to the wavelet basis. This is an ongoing project together with T. Bubba, M. Galinier, M. Lassas and S. Siltanen. The main numerical insights regarding the implementation, the design of the network and its training are provided in the related talk ‘Unrolled ISTA and convolution neural networks for limited-angle tomography reconstruction’ by M. Galinier.

Unrolled ISTA and convolutional neural networks for limited-angle tomography reconstruction

MATHILDE GALINIER (Università degli studi di Modena e Reggio Emilia)

Computed Tomography makes use of computer-processed combinations of many X-ray measurements of an object, taken from different angles, and attempts to recover the inner structure of the object from the data. In the case of limited-angle tomography, the reconstruction problem is severely ill-posed and the traditional reconstruction methods, e.g. filtered backprojection (FBP), do not perform well. In this work, we investigate a brand-new method for limited-angle tomography reconstruction, based on the unrolled version of the ISTA algorithm where each iteration contains a convolutional neural network (CNN). The idea of this project has emerged from the observation that the backprojection operator can be approximated by a sequence of convolutions applied to the original object in the wavelet domain. Thus, each CNN has the same structure as the previously mentioned sequence of convolutions.

Inverse problems and deep learning in predictive control for adaptive optics

JALO NOUSIAINEN (Lappeenranta-Lahti University of Technology LUT)

We discuss novel control algorithms in single conjugate adaptive optics (SCAO) by modern machine learning and inverse problem methods. Improvements in control have a direct impact for imaging since a significant term in the SCAO error budget is the temporal bandwidth error. This research grows out of the conviction that the vast amounts of telemetry data produced in modern AO systems can be utilized to this end. In a successful control framework predictions in the data domain or in state space aim to specify the temporal evolution of turbulence based on a recent block of data. We compare a inverse problem formulation utilizing the Taylor frozen flow model to a machine learning predictions made in the data domain.

3.2.2 Alvar 9:00 – 10:15

Detecting blockages in water supply networks using boundary control

EMILIA BLÅSTEN (University of Helsinki)

I will present a reconstruction algorithm for solving the cross-sectional area of water pipes in a network from boundary measurements. This is equivalent to reconstructing the first order perturbation to a wave equation on a quantum graph without loops from boundary measurements at all network ends except one. The method is based on a simple time reversal boundary control method originally presented by Sondhi and Gopinath for one dimensional problems and later by Oksanen to higher dimensional manifolds. This is joint work with Fedi Zouari, Moez Louati and Mohamed S. Ghi-daoui (Dept. of Civil and Environmental Engineering, The Hong Kong University of Science and Technology).

A resolvent estimate for the magnetic Schrödinger operator in the presence of short and long-range potentials

LEYTER POTENCIANO-MACHADO (University of Jyväskylä)

It is well known that the *resolvent* of the free Schrödinger operator at energy $\lambda > 0$ has norm decaying as $\lambda^{-\frac{1}{2}}$ on weighted L^2 -spaces. This is known as the *Limiting Absorption Principle*. Combining elementary techniques like integration by parts, a positive commutator argument, and a suitable Carleman estimate, we show that this result is also valid in dimension $n \geq 2$ for perturbations of the free case by *magnetic* and *electric* potentials satisfying *short* and *long-range* conditions at infinity. This is joint work with Mikko Salo (University of Jyväskylä, Finland) and Cristóbal Meroño (Universidad Politécnica de Madrid, Spain).

Determining a random Schrödinger equation

SHIQI MA (University of Jyväskylä)

We study an inverse scattering problem associated with a Schrödinger system where both the potential and source terms are random and unknown. The well-posedness of the forward scattering problem is first established in a proper sense. We then derive two unique recovery results in determining the rough strengths of the random source and the random potential, by using the corresponding far-field data. The first recovery result shows that a single realization of the passive scattering measurements uniquely recovers the rough strength of the random source. The second one shows that, by a single realization of the backscattering data, the rough strength of the random potential can be recovered. The ergodicity is used to establish the single realization recovery. The asymptotic arguments in our study are based on techniques from theory of pseudodifferential operators and microlocal analysis.

Spectral characterizations of Besse and Zoll Reeb flows

MARCO MAZZUCHELLI (École Normale Supérieure de Lyon)

A closed Riemannian manifold is called Zoll when its unit-speed geodesics are all periodic with the same minimal period. This class of manifolds has been thoroughly studied since the seminal work of Zoll, Bott, Samelson, Berger, and many other authors. It is conjectured that, on certain closed manifolds, a Riemannian metric is Zoll if and only if its unit-speed periodic geodesics all have the same minimal period. In this talk, I will first discuss the proof of this conjecture for the 2-sphere, which builds on the work of Lusternik and Schnirelmann. I will then present a stronger version of this statement valid for general Reeb flows on closed contact 3-manifolds: the closed orbits of any such Reeb flow admit a common period if and only if every orbit of the flow is closed. Time permitting, I will also summarize some related results for Reeb flows on higher dimensional contact spheres and for geodesic flows on simply connected compact rank-one symmetric spaces. The talk is based on joint works with Suhr, Cristofaro Gardiner, and Ginzburg-Gürel.

First-order primal-dual methods for non-linear inverse problems

TUOMO VALKONEN (University of Helsinki & Escuela Politécnica Nacional)

Convex optimisation problems can frequently be solved more efficiently by converting their original primal form into a dual form or saddle-point form. A popular algorithm for such problems is the primal-dual proximal splitting (PDPS) of Chambolle and Pock. Until recently, non-convex problems were most commonly solved by second-order methods in their primal form. In this talk, we discuss recent extensions of the PDPS to increasingly more complex non-smooth non-convex optimisation problems arising from non-linear inverse problems

Inverse problem for diffusion equations

YAVAR KIAN (Aix-Marseille University)

We consider the inverse problem of determining uniquely an expression appearing in a, linear or non-linear, diffusion equation. In the linear case, our equation is a convection-diffusion type of equation describing the transfer of mass, energy and other physical quantities. Our inverse problem consists in determining the velocity field associated with the moving quantities as well as information about the density of the medium. We consider this problem in a general setting where we associate the information under consideration with non-smooth coefficients depending on time and space variables. In the non-linear case, we treat the determination of a quasi-linear term appearing in a non-linear diffusion equation. This talk is based on a joint work with Pedro Caro.

3.2.4 Wivi 13:30 – 14:45

Analysis of a Dynamical System Modelling Lasers and Applications for Optical Neural Networks

LAURI YLINEN (University of Helsinki)

It is a ubiquitous phenomenon in nature that coupled oscillators tend to synchronize. The consequences range from the desirable (e.g. beating of the heart) to the catastrophic (e.g. collapsing of bridges). In this talk, we consider a dynamical system of nonlinear differential equations that models an oscillatory system (a semiconductor laser) coupled to an external driving force (an optical injection from another laser). It has been recently shown that due to synchronization of the optical fields, such a system can be used to construct a programmable all-optical logic gate. We explore mathematical properties of the system, and propose a design for an optical neural network. This is joint work with Matti Lassas and Tuomo von Lerber.

Deep convolutional neural networks for estimating porous material properties using microwave tomography

RAHUL YADAV (University of Eastern Finland)

The use of microwave tomography (MWT) to estimate porous material properties is demonstrated in this study. The studied imaging modality can be applied to estimate the moisture content distribution in a polymer foam during the microwave drying process. To solve the estimation problem related to MWT, a deep convolutional neural networks is used considering the need for real-time reconstruction. The trained neural network performance is tested with different test samples and its robustness to measurement noise is also evaluated. The imaging modality is demonstrated with synthetic measurement data for various moisture content distribution scenarios.

Learned image reconstruction for large scale tomographic imaging

ANDREAS HAUPTMANN (University of Oulu & University College London)

Recent advances in deep learning for tomographic reconstructions have shown great potential to create accurate high quality images with a considerable speed-up compared to classical reconstruction methods. This is especially true for model-based learned (iterative) reconstruction schemes. However, applicability to large scale inverse problems is limited by available memory for training and extensive training times. In this talk I will discuss applicability of learned image reconstruction approaches to tomographic data. In particular we will discuss various imaging scenarios and modalities, suitable approaches to design a robust learning task, as well as some solutions to obtain scalable learned image reconstruction for large scale and high dimensional data.

UQ for satellite measurements of CO₂ using MCMC

OTTO LAMMINPÄÄ (Finnish Meteorological Institute)

Proper monitoring and prediction of climate change and global warming requires global, satellite-based measurements of atmospheric greenhouse gas concentrations with an unprecedented accuracy. This is an ill-posed and non-linear statistical inverse problem, to which Markov Chain Monte Carlo (MCMC) methods offer a rigorous and reliable means of uncertainty quantification (UQ) and accuracy assessment. In this work, we implement adaptive MCMC together with dimension reduction to NASA's Orbiting Carbon Observatory 2 satellite's atmospheric carbon dioxide (CO₂) concentration measurements.

Uncertainty quantification for PDE-constrained optimal control problems using quasi-Monte Carlo methods

VESA KAARNIOJA (University of New South Wales)

Contemporary quasi-Monte Carlo (QMC) methods are based on crafting specially designed cubature rules for high-dimensional integration problems. Their effectiveness stems from leveraging the smoothness and anisotropy of an integrand in order to obtain faster-than-Monte Carlo convergence rates. Lately, QMC methods have become a popular tool in the study of PDEs involving random coefficients, which arise in the field of uncertainty quantification. In this work, we analyze the application of a tailored QMC method to an optimal control problem under uncertainty, where the target function is the solution of an elliptic PDE with random coefficients, steered by a control function. The robust formulation of the optimization problem is stated as a high-dimensional integration problem over the stochastic variables. It turns out that QMC methods are particularly well suited for this task: unlike, say, sparse grid methods, QMC methods preserve the convexity structure of the optimization problem. In particular, we show that under moderate assumptions on the decay of the input random field, the convergence rate obtained by using a specially designed, randomly shifted rank-1 lattice QMC rule is essentially linear, and thus superior to a Monte Carlo method. The effects of dimension truncation and finite element discretization errors are also discussed and the theoretical rates are assessed by numerical experiments. This talk is based on a joint work with Philipp A. Guth (University of Mannheim), Frances Y. Kuo (UNSW Sydney), Claudia Schillings (University of Mannheim), and Ian H. Sloan (UNSW Sydney). Reference: P. A. Guth, V. Kaarnioja, F. Y. Kuo, C. Schillings, and I. H. Sloan. A quasi-Monte Carlo method for an optimal control problem under uncertainty. Preprint, arXiv:1910.10022 [math.NA].

Wave speed uncertainty in Acousto-Electric Tomography

BJØRN CHRISTIAN SKOV JENSEN (Technical University of Denmark)

In acousto-electric tomography one needs knowledge of an acoustic wave to perform reconstruction, however, in practise this wave is very hard or potentially impossible to know exactly. In this work we consider the error severity imposed when simplistic assumptions on the wave speed yield a wrong wave resulting in a erroneous forward model. We establish theoretical bounds on the error in the electrical field with respect to the error in wave speed and perform simulations of the phenomenon.

3.2.6 Wivi 15:15 – 16:05

The geodesic X-ray transform on disks of constant curvature

FRANÇOIS MONARD (University of California, Santa Cruz)

We will review recent results on the study of the geodesic X-ray transform on disks of constant curvature. This is a two-parameter family of simple Riemannian surfaces whose limiting cases attain certain borderline cases of simplicity. For such a family, one can formulate rather explicit statements on the mapping properties of the geodesic X-ray transform and the associated normal operator, its singular value decomposition and its range characterization. Some works presented are joint with Rohit Mishra (UT Arlington).

Classification of the stroke using Neural Networks in Electrical Impedance Tomography

RASHMI MURTHY (University of Helsinki)

Electrical impedance tomography (EIT) is an imaging method based on probing an unknown conductive body with electrical currents. Voltages resulting from the current feeds are measured at the surface, and the conductivity distribution inside is reconstructed. This is a promising technique in medical imaging as various organs and tissues have different conductivities. The motivation of this talk arises from imaging strokes in the brain. However, the example presented is a simplified model aimed at demonstrating the benefits of Neural Networks. In this talk we present the results of the classification of the two different kinds of stroke using Neural networks that are fed data that has been preprocessed with the nonlinear inverse techniques.

3.3 Day 3, Wednesday 18

3.3.1 Wivi 9:00 – 10:15

Gray-box machine learning for electrical impedance tomography

SAMULI SILTANEN (University of Helsinki)

Electrical impedance tomography is an imaging method based on probing an unknown conductive body with electrical currents. Voltages resulting from the current feeds are measured at the surface, and the conductivity distribution inside is reconstructed. This is a promising technique in medical imaging as various organs and tissues have different conductivities. The motivation of this talk arises from imaging strokes in the brain. However, the example presented is a simplified model aimed at demonstrating the benefits of gray-box machine learning (ML) over black-box ML. Namely, we demonstrate that the convex hull of inclusions in homogeneous background conductivity can be recovered better with a combination of Ikehata's enclosure method and convolutional neural networks (CNNs) than with either of those methods alone. Therefore, traditional regularized inversion methods can be used as nonlinear features for ML.

Non-linear difference reconstruction method for monitoring of cerebral haemorrhage

VILLE KOLEHMAINEN (University of Eastern Finland)

We present a non-linear difference imaging algorithm for monitoring of progression of cerebral haemorrhage using EIT. The method is based on a joint reconstruction formulation where the initial conductivity and the change in the conductivity are reconstructed simultaneously based on the voltage data before and after the change.

Deep Learning based Virtual Hybrid Edge Detection Method in Electrical Impedance Tomography for Stroke Classification

AYNUR CÖL (Sinop University)

Electrical Impedance Tomography (EIT) is a type of medical imaging in which the electrical conductivity is reconstructed inside an unknown physical body from measurements of voltages and currents at the boundary. Although the exponential ill posedness in EIT, the recent method of Greenleaf et al. is a robust method called virtual hybrid edge detection (VHED) in two dimensions to determine the singularities of a conductivity from EIT data. In this work, the data is created with the help of the computational studies of VHED method for stroke types and the classification of strokes is examined by using artificial neural network (ANN).

3.3.2 Alvar 9:00 – 10:15

An inverse problem for the fractional Schrödinger equation in a magnetic field

GIOVANNI COVI (University of Jyväskylä)

In this talk I will show global uniqueness in an inverse problem for a fully fractional magnetic Schrödinger equation (FMSE): an unknown electromagnetic field in a bounded domain is uniquely determined up to a natural gauge by infinitely many measurements of solutions taken in arbitrary open subsets of the exterior. The proof is based on Alessandrini's identity and the Runge approximation property, thus generalizing some previous works on the fractional Laplacian.

An inverse problem for a generalized fractional derivative with an application in reconstruction of time- and space-dependent sources in fractional diffusion and wave equations

NATALIJA KINASH (Tallinn University of Technology)

In this presentation, we consider two inverse problems with a generalized fractional derivative. The first problem IP1 is to reconstruct the function u based on its value and the value of its fractional derivative in the neighbourhood of the final time. We prove the uniqueness of solution to this problem. Afterward we investigate the IP2 that is to reconstruct a source term in an equation that generalizes fractional diffusion and wave equations, given measurements in a neighbourhood of final time. The source to be determined depends on time and all space variables. The uniqueness is proved based on the results for IP1. Finally, we derive the explicit solution formulas to the IP1 and IP2 for the particular cases of the generalized fractional derivative. These are usual fractional derivative, tempered and Atangana-Baleanu fractional derivatives.

Nonlinear Lavrentiev regularization of monotone ill-posed equations

FREDRIK HILDRUM (Norwegian University of Science and Technology)

In this talk we will discuss a subgradient-based variant of Lavrentiev regularization of the form

$$\mathcal{A}u + \alpha \partial \mathcal{R}(u) \ni f^\delta$$

for monotone operators \mathcal{A} in Hilbert spaces, in which the subdifferential replaces the identity operator as the regularization term. In contrast to Tikhonov regularization, this approach avoids evaluation of the adjoint operator \mathcal{A}^* . Consequently, it can be used, for instance, for the solution of Volterra integral equations of the first kind-where the adjoint would require an integration forward in time-without the need of accessing future data. We will address well-posedness and convergence rates for this method, and briefly touch upon a fast numerical algorithm for total variation-denoising of one-dimensional piecewise constant signals. *This is joint work with Markus Grasmair, NTNU.*

3.3.3 Wivi 10:45 – 12:00

Monotonicity-based reconstruction of extreme inclusions in electrical impedance tomography

VALENTINA CANDIANI (Aalto University)

The monotonicity-based approach has become one of the fundamental methods for reconstructing inclusions in the inverse problem of electrical impedance tomography. Thus far the method has not been proven to be able to handle extreme inclusions that correspond to some parts of the studied domain becoming either perfectly conducting or perfectly insulating. The main obstacle has arguably been establishing suitable monotonicity principles for the corresponding Neumann-to-Dirichlet boundary maps. In this work, we tackle this shortcoming by first giving a convergence result in the operator norm for the Neumann-to-Dirichlet map when the conductivity coefficient decays to zero and/or grows to infinity in some given parts of the domain. This allows passing the necessary monotonicity principles to the limiting case. Subsequently, we show how the monotonicity method generalises to the definite case of reconstructing either perfectly conducting or perfectly insulating inclusions, as well as to the indefinite case where the perturbed conductivity can take any values between, and including, zero and infinity.

Shape-based image reconstruction of electrical/ultrasonic dual-modality tomography

GUANGHUI LIANG (Tianjin University & University of Eastern Finland)

Electrical impedance tomography (EIT) and ultrasonic reflection tomography (URT) have been widely used in industrial and biomedical fields for the advantages of non-invasive, low-cost, and portable. However, the achievable imaging resolution and noise robustness of single-modality EIT or URT method are usually limited, where EIT suffers from the nonlinear ill-posed inverse problem and URT faces the problem that high precision means more measurements and complex calculation processing. Inspired by multimodal information fusion, we have made some attempts to improve the imaging quality by combining electrical/ultrasonic dual-modality measurement information. In order to construct the joint reconstruction model, the reconstruction goals of different modalities are unified into dimensionless shapes of the targets. It is reasonable in some applications that the reconstructed targets satisfy piecewise-constant distribution and the sensitive physical parameters of different modalities have the distribution consistency. Thus, the unknowns to be reconstructed of EIT and URT will be transformed into the shape parameters characterizing the shape of the target, which can also reduce the number of the unknowns. Then, the augmented Lagrange method is used to construct the EIT/URT dual-modality inverse problem, and the optimal shape parameters can be calculated by the Gauss-Newton iteration method. The numerical and experimental tests have been carried out, and the results show that the proposed EIT/URT dual-modality tomography method has better imaging accuracy and noise robustness than the traditional single-modality tomography method.

Optimal depth-dependent distinguishability bounds for EIT

HENRIK GARDE (Aalborg University)

The talk is on the inverse problem of electrical impedance tomography (EIT). Distinguishability of inclusions is on determining estimates of the norm-difference of two measurements of data (Dirichlet-to-Neumann maps); one measurement with and one without an inclusion. This indicates the amount of noise that is needed to make such an inclusion invisible for numerical reconstruction. In this talk I will present some depth-dependent distinguishability bounds for EIT, that depend on the distance of a ball inclusion to the boundary of a ball-domain. The bounds are optimal for the limit-cases of infinitesimal and of domain-covering inclusions in any dimension.

3.3.4 Alvar 10:45 – 12:00

Imaging of moisture flows in cement-based materials using X-ray computer tomography and electrical capacitance tomography

PETRI KUUSELA (University of Eastern Finland)

Quantifying moisture transport properties of cement-based materials is important, because their durability depends on how rapidly moisture and aggressive agents ingress into the materials. Because current techniques to characterize the properties of these materials are limited, new tools are needed. In this talk, we show results of experimental studies, which investigated the capability of two tomographic modalities--X-ray CT and electrical capacitance tomography--to image 3D moisture flows in cement-based materials.

Modelling of errors due to uncertainties in ultrasound sensor locations in photoacoustic tomography

TEEMU SAHLSTRÖM (University of Eastern Finland, Department of Applied Physics)

Photoacoustic tomography (PAT) is a biomedical imaging modality developed during the past decades. In PAT, the imaged target is illuminated with a short light pulse. Absorption of this light is followed by local thermal expansion and mechanical stress resulting in an initial pressure p_0 within the target. This initial pressure relaxes as broadband ultrasound waves that are recorded on the boundary of the imaged target. In the inverse problem of PAT, the aim is to reconstruct p_0 given the measured time-pressure series p_t [1]. PAT combines unique optical contrast with high ultrasound resolution, and it has found applications for example in imaging tissue vasculature and small animal imaging. To produce accurate photoacoustic images, the underlying physics and measurement setup parameters have to be modelled to a sufficient accuracy. In practice, the model used in the inverse problem of PAT can, however, include various modelling errors such as errors due to coarse numerical discretisation, simplifications of the numerical model or uncertainties in model parameters. Due to the ill-posedness of the inverse problem, the modelling errors can result in significant errors in the solution of the inverse problem. In this work, we study the effect and compensation of modelling errors due to uncertainties in ultrasound sensor locations in PAT [2]. The studied geometry is composed of ultrasound sensors that are located on a circle with varying degrees of radial and angular uncertainties. The inverse problem of PAT is approached in a Bayesian framework. Compensation of the modelling errors is carried out using the Bayesian approximation error modelling [3]. The approach is evaluated with simulated and experimental data using multiple sensor geometries and levels of uncertainty. The results indicate that the inverse problem of PAT is sensitive even to small uncertainties in ultrasound sensor locations. It was found that uncertainty in radial locations resulted in larger modelling errors than similar uncertainty in angular positions. Furthermore, the errors in the solution of the inverse problems are most significant when using limited-view measurement geometries. These errors due to uncertainties in ultrasound sensor locations can, however, be effectively compensated using the Bayesian approximation error modelling. This is joint work with Aki Pulkkinen, Jenni Tick, Jarkko Leskinen and Tanja Tarvainen.

References:

- [1] L. Wang and J. Yao “A practical guide to photoacoustic tomography in life sciences,” *Nature Methods*, 13(8):627–638, 2016.
- [2] T. Sahlström, A. Pulkkinen, J. Tick, J. Leskinen and T. Tarvainen, “Modelling of errors due to uncertainties in ultrasound sensor locations in photoacoustic tomography,” *IEEE Trans Med Imag*, submitted, 2019.
- [3] J. Kaipio and E. Somersalo, *Statistical and Computational Inverse Problems*. New York: Springer, 2005.

Simultaneous Reconstruction of Emission and Attenuation in Passive Gamma Emission Tomography of Spent Nuclear Fuel

RASMUS BACKHOLM (University of Helsinki)

The International Atomic Energy Agency (IAEA) has recently approved passive gamma emission tomography (PGET) as a method for inspecting spent nuclear fuel assemblies (SFAs), an important aspect of international nuclear safeguards. The PGET instrument is essentially a single photon emission computed tomography (SPECT) system that allows the reconstruction of axial cross-sections of the emission map of the SFA. The fuel material strongly self-attenuates its gamma-ray emissions, so that correctly accounting for the attenuation is a critical factor in producing accurate images. Due to the nature of the inspections, it is desirable to use as little a priori information as possible about the fuel, including the attenuation map, in the reconstruction process. Current reconstruction methods either do not correct for attenuation, assume a uniform attenuation throughout the fuel assembly, or assume an attenuation map based on an initial filtered back projection reconstruction. Here, we propose a method to simultaneously reconstruct the emission and attenuation maps by formulating the reconstruction as a constrained minimization problem with a least squares data fidelity term and regularization terms. The performance of the proposed method, with two different regularizers, is evaluated with simulated data that includes missing rods and rods replaced with fresh fuel. The method is shown to produce good results when comparing the reconstructions to the ground truth by various numerical metrics, and when classifying the rods with the method currently employed by the IAEA. Additionally, the proposed method is shown to allow for an enhanced classification method that uses also the reconstructed attenuation map.

3.3.5 Wivi 13:15 – 14:05

Infinite-dimensional inverse problems with a finite number of measurements

MATTEO SANTACESARIA (University of Genoa)

In this talk I will discuss how ideas from applied harmonic analysis, in particular sampling theory and compressed sensing, may be applied to inverse problems for partial differential equations. The focus will be on inverse boundary value problems for the conductivity and the Schrodinger equations, but the approach is very general and allows to handle many other classes of inverse problems. I will discuss uniqueness, stability and reconstruction, both in the linearized and in the nonlinear case. This is joint work with Giovanni S. Alberti.

Developments on the logarithmic forward map of electrical impedance tomography

TOPI KUUTELA (Aalto University)

In this talk, we consider regularity properties of the completely logarithmic forward map of electrical impedance tomography (EIT), i.e. a mapping from logarithm of the conductivity to the logarithm of the Neumann-to-Dirichlet map of the conductivity equation. In particular, we prove the continuous Fréchet differentiability of the completely logarithmic forward map of EIT and take a look into some natural corollaries. Furthermore, we consider how the mapping could be generalized to "non-symmetric" EIT problem, problem in which the Neumann and Dirichlet boundary conditions are defined on distinct sections of the boundary. This talk is based on joint work with Henrik Garde and Nuutti Hyvönen.

3.3.6 Alvar 13:15 – 14:05

Generic uniqueness and stability for mixed ray transform

TEEMU SAKSALA (Rice University)

We consider the mixed ray transform of tensor fields on a three-dimensional compact simple Riemannian manifold with boundary. We state an elastic travel-time tomography problem for qS -waves and recall how the linearization of this problem leads to mixed ray transform for $2 + 2$ tensor fields. We prove the injectivity of the transform, up to natural obstructions, and stability estimates for the normal operator on generic three dimensional simple manifold in the case of $1 + 1$ and $2 + 2$ tensors fields. These are the first steps towards solving the elastic travel time tomography problem. Talk is based on joint work with: Maarten V. de Hoop, Gunther Uhlmann and Jian Zhai.

1D source detection with multiple frequencies

TOMMI BRANDER (Norwegian University of Science and Technology)

We study an inverse problem where an unknown radiating source is observed with collimated detectors along a single line and the medium has a known attenuation. The research is motivated by applications in SPECT and beam hardening. If measurements are carried out with frequencies ranging in an open set, we show that the source density is uniquely determined by these measurements up to averaging over levelsets of the integrated attenuation. This leads to a generalized Laplace transform. We also discuss numerical approaches and present reconstructions. Joint work with Joonas Ilmavirta and Teemu Tyni.

The light ray transform in stationary and static Lorentzian geometries

LAURI OKSANEN (University College London)

Given a Lorentzian manifold, the light ray transform of a function is its integrals along null geodesics. We consider injectivity of the light ray transform on functions and tensors, up to the natural gauge for the problem. First, we study the injectivity of the light ray transform of a scalar function on a globally hyperbolic stationary Lorentzian manifold and prove injectivity holds if either a convex foliation condition is satisfied on a Cauchy surface on the manifold or the manifold is real analytic and null geodesics do not have cut points. Next, we consider the light ray transform on tensor fields of arbitrary rank in the more restrictive class of static Lorentzian manifolds and show that if the geodesic ray transform on tensors defined on the spatial part of the manifold is injective up to the natural gauge, then the light ray transform on tensors is also injective up to its natural gauge. Finally, we provide applications of our results to some inverse problems about recovery of coefficients for hyperbolic partial differential equations from boundary data.

New deep neural networks solving non-linear inverse problems

MATTI LASSAS (University of Helsinki)

We present the construction of a new type of deep neural network developed to solve nonlinear inverse problems for wave equations. In particular, we consider the determination of an unknown wave speed on an interval from boundary measurements, where wave propagation is governed by the acoustic wave equation. A novel feature of the studied neural network is that the data itself form layers in the network. Using the theory of classical inverse problems we can justify the architecture and analyse the properties of these neural networks. For inverse problems, the main theoretical questions concern uniqueness, range characterisation, stability and the regularisation strategies for the inverse problems. We will discuss the question when a solution algorithm generalises from the training data, that is, when the solution algorithm trained with a finite number of samples can solve the problem with new inputs that are not contained in the training data. This can be viewed as a new question for classical inverse problems that takes its motivation from machine learning. The results are done in collaboration with Christopher A. Wong and Maarten de Hoop.

4 POSTER ABSTRACTS

Estimation of dynamic SNP-heritability with Bayesian Gaussian process models

ARTTU ARJAS (University of Oulu)

Improved DNA technology has made it practical to estimate single nucleotide polymorphism (SNP)-heritability among distantly related individuals with unknown relationships. For growth and development related traits, it is meaningful to base SNP-heritability estimation on longitudinal data due to the time-dependency of the process. However, only few statistical methods have been developed so far for estimating dynamic SNP-heritability and quantifying its full uncertainty. We introduce two Bayesian Gaussian process (GP) based approaches for estimating dynamic variance components and heritability as their function. For parameter estimation, we use modern Markov Chain Monte Carlo (MCMC) methods which allow full uncertainty quantification. Several data sets are analysed and our results clearly illustrate that the 95 % credible intervals of the joint estimation method (which “borrows strength” from adjacent time points) are significantly narrower than of the two-stage independent modelling method. On the other hand, the two-stage method is faster and scales better when the dimension of the data increases.

Flat-Field Correction in Absorption Tomography

KATRINE OTTESEN BANGSGAARD (Technical University of Denmark)

Absorption tomography is a non-invasive scanning technique used to acquire morphological information of the interior of objects, without cutting or damaging them. In absorption tomography most reconstruction methods are based on the assumption that the detector response is known. Unfortunately, this is not always the case. In practice the detector response must be estimated by flat-field measurements which are measurements in the scanner without an object. The flat-field measurements are often uncertain due to the statistical nature of photons and the flat-field estimation may introduce ring artifacts in the reconstruction as a consequence. We propose a new convex reconstruction model that simultaneously estimates the reconstruction and detector response by modeling the statistical nature of the measurements. We compare the reconstruction obtained by our model with reconstructions obtained by ring reducing preprocessing methods. Discrepancies between the true and estimated detector response are especially problematic when the acquisition time or the dose is limited. Preliminary experiments show that the proposed reconstruction model leads to reduction of systematic errors for dose-limited absorption tomography.

Physical model of tumbling asteroid 2012 TC4

JOSEF DURECH (Astronomical Institute, Faculty of Mathematics and Physics, Charles University)

Shapes and spin states of asteroids can be reconstructed from their disk-integrated time-resolved photometry by the lightcurve inversion method of Kaasalainen et al. (2001, Icarus 153, 37). The rotation of asteroids that are not in the minimum-energy rotation state - so-called tumbling asteroids - can be described as free precession and the rotation parameters can be also uniquely reconstructed (Kaasalainen 2001, A&A 376, 302). We will present a physical model of small (10 m) near-Earth asteroid 2012 TC4 reconstructed from its lightcurves obtained during its close Earth fly-by in October 2017. To derive the best-fit model, we scanned a wide range of parameters using a model of a geometrically scattering ellipsoid. In this approach, the dynamical parameters (moments of inertia) describing the excited rotation are always consistent with the shape, which cannot be automatically fulfilled when using convex shape models. Moreover, the brightness of an ellipsoidal model for a given orientation can be computed analytically, which speeds up the optimization. The dynamical parameters found with the ellipsoidal model were then used for the final optimization with a convex shape model.

Sparse dynamic tomography: a shearlet based approach for iodine perfusion in plant stems

TOMMI HEIKKILÄ (University of Helsinki)

Dynamic x-ray tomography is an extension of the well known ill-posed inverse problem of reconstructing the inner structure of a target from x-ray data collected from multiple directions or projections. In dynamic tomography the target is non-stationary which makes many traditional methodologies unsuitable if the movement is not in some way compensated or negated. Our approach is to consider a sparse set of measurements repeated over time to obtain data with two spatial dimension and a temporal dimension. The problem is then presented as a set of linear inverse problems corresponding to a minimization problem of the data mismatch and regularization. Here we use shearlets, a modern multivariate representation system, iterative primal-dual fixed point algorithm and a recently proposed method for automated choice of regularization parameter. The motivation of this work stems from x-ray imaging of plants perfused with a liquid contrast agent aimed at increasing the contrast of the images and studying the flow of liquid in the plant over time. Therefore the task is to obtain dynamic tomographic reconstructions. The main idea is to apply 3D shearlets as a prior, treating time as the third dimension. For comparison, 2D Haar wavelet transform and a 2D shearlet transform were also tested. The quality of different set-ups were assessed for said problem with simulated measurements, a real life scenario where the contrast agent is applied to a gel and, finally, to real data where the contrast agent is perfused to a real plant. The results indicate that the 3D shearlet-based approach produce suitable reconstructions for observing the changes in the contrast agent even though there are no drastic improvements to the quality of reconstructions compared to using the simpler Haar transform. This is a joint work between the Department of Physics and the Department of Mathematics and Statistics in the University of Helsinki.

Optical projection tomography for tissue engineering and its mathematical modeling to improve reconstruction quality

OLLI KOSKELA (Häme University of Applied Sciences)

Optical projection tomography (OPT) is a powerful technique for imaging mesoscopic samples that have diameter of 1–10 mm. As the name suggest, OPT is based on propagating light through transparent or semi-transparent medium and collecting these projection images in series of angular position while rotating the sample. Similar to X-ray computed tomography (CT), from projections of attenuated light it is possible to reconstruct the attenuation coefficient distribution within the sample. Since light has almost negligible effect on living tissue, OPT is a super candidate for long term tissue engineering and cell culture imaging. The optical contrast in such samples is good, thus further motivating the use of OPT. Among others, OPT has been successfully applied in the studies of mice pancreas and brain, as well as cell and tissue imaging in hydrogel cultures. OPT can also be used in fluorescence mode, where emission light sources inside the sample are excited from the outside. As a reconstruction method in OPT, the golden-standard from X-ray CT, filtered backprojection (FBP) with parallel beam assumption, has been used with surprisingly good results--given that the optics used in OPT collect focused light where features are in focus only on certain distance from the detection objective. Detected features become blurrier the further they are from the objective. When the sample rotates different portions of the sample are in focus at different projections. However, FBP assumes absolute focus throughout the sample. In our work, we strive to increase the resolution from micrometers to sub-micrometer through technical improvements and mathematical modeling. This presentation concentrates in the latter. We show how Gaussian light beam modeling can be used with brightfield mode OPT and, furthermore, an adaptation of X-ray fluorescence reconstruction method to fluorescence mode OPT. With both modes, the presented mathematical model is an improvement when compared with FBP, in terms of both the physical accuracy and reconstruction quality. The performance is demonstrated with experimental data from zebrafish and cell-mimicking optical beads. Also, we provide open data sets and MATLAB (The MathWorks, Inc.) code files related to our articles.

Modified space time level set method in dynamic tomography

SALLA LATVA-ÄIJÖ (University of Helsinki)

Salla-Maaria Latva-Äijö University of Helsinki, Finland salla.latva-aijo@helsinki.fi Dynamic tomography is of great interest in modern X-ray tomography, mainly because imaging of dynamic systems is a challenge for medical CT-imaging. Level set (LS) method algorithms are in wide use in the inversion and segmentation of data. The modified level set (MLS) method [1] is a reconstruction technic, which is able to pick the greatest changes from the image, for example boundaries between different materials. Because MLS method does regularization in both spatial and temporal direction, it works well with time-dependent targets. We tested the MLS reconstruction approach with a self-made time-dependent phantom and got good results even with extremely sparse data. Our next aim is to combine MLS method to multi-energy X-ray tomography. This is a way to increase the number of information available from one X-ray-scan. Materials attenuate X-rays differently, depending on the energy of the X-ray spectrum used. By using several level set functions in reconstructions, we can differentiate several materials and use the mathematical model to determine the amount of materials in the target. [1] Niemi E, Lassas M, Kallonen A, Harhanen L, Hämäläinen K and Siltanen S 2015, Dynamic multi-source X-ray tomography using a spacetime level set method. *Journal of Computational Physics* 291, pp. 218-237. Keywords: dynamic tomography, level set methods, multi-energy tomography

Finite element model of a human head for simulated multi-frequency electrical impedance tomography measurement results

ANTTI PALDANIUS (Tampere University)

Currently real-time monitoring of progression of intracerebral hemorrhage (ICH) is difficult. The main imaging method for ICH, computed tomography (CT), is cumbersome as ICH patients are often connected to life support systems and need to be moved from intensive care unit to the CT scanner. One proposed solution is monitoring the progression of the ICH with electrical impedance tomography (EIT) by detecting changes in the electrical conductivity of the brain. The aim of this work is to produce accurate computational model for simulation of EIT measurements in order to provide simulated measurement data from various disease states to assess the sensitivity of the method and to provide simulated measurement data to people working on the inverse problem of EIT. Finite element of a human head was created from segmented data and imported into a COMSOL. The model has seven different domains included; scalp, skull, cerebrospinal fluid (CSF), grey matter, white matter, cerebellum and an ICH lesion. The model consists of 2.5 million tetrahedral elements and has complete electrode model (CEM) and frequency dependent material properties implemented. The model produces simulated voltage measurement data from electrodes placed on the scalp with different current injection patterns. The model also enables visualization of sensitivity distributions of different current injection and measurement patterns.

Periodic Radon transforms and computed tomography

JESSE RAILO (University of Jyväskylä)

We discuss our recent results on the periodic d -plane Radon transforms (arXiv:1906.05046, arXiv:1909.00495). These transforms have applications in the study of the usual Radon transforms in the Euclidean space. The X-ray transform is a special case of the Radon transform, its mathematical analysis has led to the celebrated computed tomography (CT) imaging method, and these integral transforms have been studied more than 100 years. The periodic d -plane Radon transform is an analog of the usual d -plane Radon transform acting on periodic functions on the flat tori. It is an operator that encodes the integrals of a function over periodic d -planes. The associated inverse problem is to determine a function from this data. We present new reconstruction formulas, stability results and Tikhonov regularization methods to this problem in Sobolev spaces. The two dimensional case (including simulations) is joint work with Joonas Ilmavirta (Jyväskylä) and Olli Koskela (Tampere).

Acousto-electric tomography on Riemannian manifolds

HJØRDIS AMANDA SCHLÜTER (Technical University of Denmark)

Acousto-electric tomography (AET) is an imaging modality that augments electrical impedance tomography (EIT) with acoustic waves to reconstruct the interior electrical conductivity in an object from exterior measurements. Due to the additional information from the acoustic waves one is able to obtain interior power density data, and from this data the conductivity can be reconstructed with high resolution compared to EIT. We aim at addressing the AET problem in a general setting for anisotropic conductivities on general Riemannian manifolds. This research is motivated by similar studies for EIT, where lifting the problem to a general, geometric setting has given new insight. We illustrate our initial results with a simple example concerning an isotropic conductivity in a domain on a catenoid.

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