

Inverse problems, PDE and geometry

University of Jyväskylä, Finland

August 20–23, 2018

Conference information

Inverse problems, PDE and geometry

The workshop "Inverse problems, PDE and geometry" focuses on recent progress in the mathematical theory of inverse problems and related methods in PDE, geometry and microlocal analysis. You are warmly welcome in Jyväskylä!

Scientific committee:

- Yaroslav Kurylev (UCL)
- Mikko Salo (Jyväskylä)
- Gunther Uhlmann (Washington / HKUST)

Local organizing committee (University of Jyväskylä):

- Giovanni Covi
- Miia Honkonen
- Joonas Ilmavirta
- Jere Lehtonen
- Keijo Mönkkönen
- Leyter Potenciano
- Mikko Salo

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- European Research Council (ERC), under the European Union's Horizon 2020 research and innovation programme (ERC Consolidator Grant 770924)
- Academy of Finland / Centre of Excellence in Inverse Modelling and Imaging
- Finnish Academy of Science and Letters, Mathematics Fund
- University of Jyväskylä
- City of Jyväskylä

Conference webpage: <http://r.jyu.fi/rBP>

1 Program

	Monday 20 building: Agora	Tuesday 21 building: Agora	Wednesday 22 building: MaD	Thursday 23 building: MaD
09:00–09:45	Registration (from 09:00) Opening (09:30–09:45)	Oksanen: <i>Inverse problem for Einstein-scalar field equations</i>	Faraco: <i>Conformal invariance and limiting Carleman weights</i>	Liu: <i>Plasmon resonances in optics and elasticity and their applications</i>
09:45–10:30	Uhlmann: <i>Inverse problems in non-linear wave propagation</i>	Ivanov: <i>On distance difference representations of Riemannian manifolds</i>	Krupchyk: <i>Eigenvalue bounds for Schrödinger operators with complex potentials and non-trapping metrics</i>	Monard: <i>Attenuated tensor tomography on the Euclidean disk and the inverse source problem in transport</i>
10:30–11:00	Coffee	Coffee and posters	Coffee	Coffee
11:00–11:45	Isozaki: <i>Spectral and inverse scattering theory on graphen</i>	Stefanov: <i>Local and global boundary rigidity</i>	Paternain: <i>Convex projective surfaces with compatible Weyl connection are hyperbolic</i>	Greenleaf: <i>Microlocal analysis of Doppler synthetic aperture radar</i>
11:45–12:30	Koch: <i>Conserved energies for Gross-Pitaevskii</i>	Vasy: <i>Global analysis for linear and nonlinear waves and the stability of Kerr-de Sitter space</i>	de Hoop: <i>Unique recovery of faults and dislocations from coseismic displacement data</i>	Lassas: <i>Geometric inverse problems with random and unknown sources</i>
12:30–14:00	Lunch	Lunch	Lunch	Lunch
14:00–14:30	Balehowsky: <i>Recovering a Riemannian metric from least-area data</i>	Lefeuvre: <i>The marked length spectrum of negatively-curved manifolds</i>	Blåsten: <i>Inverse problems with one measurement</i>	
14:30–15:00	Cekić: <i>Calderón problem for Yang-Mills connections</i>	Chen: <i>Uniform Sobolev estimates on real hyperbolic spaces</i>	Feizmohammadi: <i>Uniqueness of a potential from boundary data in locally conformally transversally anisotropic geometries</i>	
15:00–15:30	Liimatainen: <i>The Poisson embedding approach to the Calderón problem</i>	Ilmavirta: <i>Functions of constant X-ray transform</i>	Lin: <i>On localizing and concentrating electromagnetic fields</i>	
15:30–16:00	Coffee	Coffee (15:30–16:00) and posters (15:30–17:00)	Coffee	
	Reception 18:00– (from Alba 17:45) Alvar Aalto Museum	Dinner 18:15– (from Alba 17:30) Savutuvan apaja		

2 Practical information

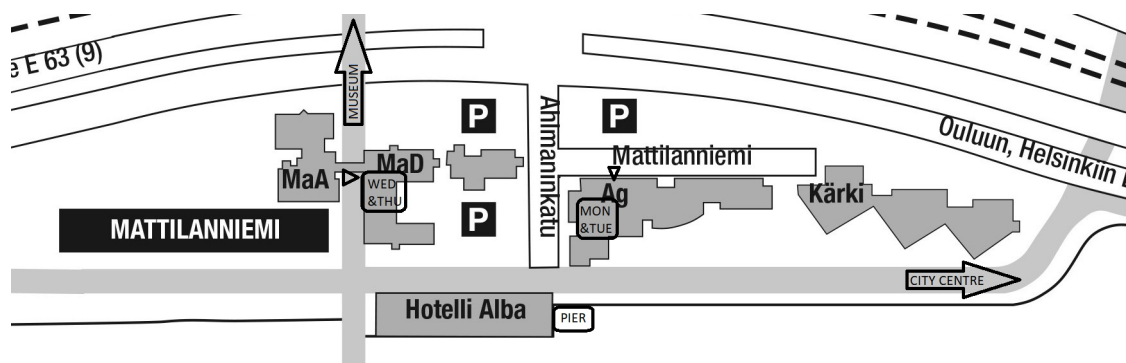
2.1 Registration

A registration desk can be found at the conference venue (building Agora on Monday and Tuesday). Registration begins on Monday at 9:00, and the opening will be at 9:30. If you have any questions during the conference, please contact the registration desk or any of the local organizers wearing an orange name tag.

2.2 Venue

On Monday and Tuesday the conference will take place in the building Agora, and on Wednesday and Thursday in the building MaD. Both buildings are located in the Mattilanniemi campus right next to hotel Alba, and are open at least from 8:00 to 18:00 on conference days. The lecture halls are the Martti Ahtisaari auditorium (also known as Agora auditorium 1) and the room MaD202.

The following map shows the two buildings, their main entrances, the pier for the dinner transportation, and directions to the city centre and the Alvar Aalto Museum. A more detailed map is available on the conference webpage.



Rooms have been reserved for collaboration or other work. Agora Beeta (AgB121.1) is available 9:00–17:00 on Monday and Tuesday, MaD381 9:00–18:00 on Wednesday and Thursday. Please ask the locals for directions if you have trouble locating the rooms.

2.3 Internet

Eduroam is available. In addition, you may use the local WiFi network with the following login details, valid August 19–24: [redacted from online version]

2.4 Lunch and coffee

Lunch is available in restaurant Piato in Agora on Monday and Tuesday and Maija in MaA on Wednesday and Thursday. For participants receiving financial support from the conference, the conference folder includes four lunch vouchers valid for a basic lunch, including salad, bread, drinks and one of the main courses on the daily menu (excluding grill). Coffee and dessert are available but not included in the vouchers. The vouchers are only valid in the specified restaurants.

Coffee breaks (including snacks) will be served near the lecture halls. The posters will be presented during the coffee breaks on Tuesday.

2.5 Reception

The city of Jyväskylä has invited us to the Alvar Aalto Museum that showcases the work of Alvar Aalto, the most prominent architect in Finland. Beside a city reception, the visit will include an optional tour of the museum. Light food will be served.

The reception starts at 18:00 on Monday. One of the organizers will leave from the Agora lobby at 17:45, and walk to the museum via hotel Alba and MaD. You may go to the museum on your own if you prefer so.

The reception is free of charge to all registered participants.

2.6 Dinner

The conference dinner will be served at Savutuvan apaja in a traditional Finnish lake environment. We will take a boat across lake Päijänne from a pier next to hotel Alba at 17:30 on Tuesday, and the event at Savutuvan apaja will begin at about 18:15. If you prefer to not use the boat, land transportation can be arranged; in that case contact the organizers in advance.

The dinner and transportation is free of charge to all registered participants.

2.7 Posters

Posters will be presented during the coffee breaks on Tuesday. As there are no events immediately after the second coffee break, the poster session is free to continue until about 17:00. Poster abstracts are included in document after the talk abstracts. Those presenting a poster are requested to bring their posters on Monday for preparations.

2.8 Local transportation

The conference venue and Hotel Alba are located 2 km from the centre and train station. Taxis are available at the taxi rank at the train station, or can be reserved by calling +358 100 6900. The organizers would also be happy to arrange a taxi for you. It is possible to walk from the centre to the conference venue along the Jyväsjärvi lake.

2.9 Local activities

For non-scientific activities in Jyväskylä, please see <https://visitjyvaskyla.fi/en>. Most museums have free admission on Fridays.

The closest restaurants from the conference site are Alba (in the hotel), Sohwi, and Morton. Some other local restaurants worth considering are Pöllöwaari, Figaro, Harmoon, Taikuri, and Harald. For tea we recommend Teeleidi close to the conference site, and there are a number of coffee places in the centre. Please contact the local organizers for further information.

3 Talk abstracts

3.1 Monday

Inverse problems in non-linear wave propagation

GUNTHER UHLMANN (University of Washington & HKUST)

We will describe a general method to solve inverse problems arising in non-linear wave propagation. In particular this method can be applied to Einstein's equations coupled with matter fields, the Einstein-Maxwell equations and also inverse problems for non-linear elastic materials.

Spectral and inverse scattering theory on graphen

HIROSHI ISOZAKI (Tsukuba University)

We consider the forward and inverse scattering problems on perturbed periodic graphs. A physically important example is the graphen, for which there are two mathematical models. The first one (the discrete model or the vertex model) deals with the propagation of waves restricted on vertices of hexagonal lattices, and the second one (the quantum graph or the edge model) describes the waves governed by the 1-dimensional Schroedinger equation on the edges. These two models are closely related. The continuous spectrum of the latter inherits from that of the former. We develop the spectral theory for these graph Laplacians by constructing a complete system of generalized eigenfunctions, studying their behavior at infinity and defining the S-matrix. The main aim is to solve the inverse scattering problem. Assuming that perturbations are confined to a finite part of the graph, we show that for the vertex mode

- (1) the S-matrix of a fixed energy determines the potential,
- (2) the S-matrix of a fixed energy determines the convex hull of defects of the lattice,
- (3) the S-matrices for all energies determines the perturbation as a planar graph.

For the edge model we show that

- (4) the S-matrices for all energies determine the potentials on all edges provided they are symmetric with respect to the center of the edge.

This is a joint work with K. Ando, E. Korotyaev and H. Morioka.

Conserved energies for Gross-Pitaevskii

HERBERT KOCH (University of Bonn)

The Gross-Pitaevskii equation is a variant of the cubic NLS with the boundary condition $|u(x)| \rightarrow 1$ as $|x| \rightarrow \infty$. There exists a Lax pair and conserved energies are closely related to the spectral properties of the Lax operator. I will explain current work with X. Liao where we construct a continuous family of conserved energies. A byproduct is global well-posedness for rough initial data.

Recovering a Riemannian metric from least-area data

TRACEY BALEHOWSKY (University of Helsinki)

In this talk, we address the following question: Given any simple closed curve γ on the boundary of a Riemannian 3-manifold (M, g) , suppose the area of the least-area surfaces bounded by γ are known. From this data may we uniquely recover g ?

In several settings, we show the the answer is yes. In fact, we prove both global and local uniqueness results given least-area data for a much smaller class of curves on the boundary. We demonstrate uniqueness for g by reformulating parts of the problem as a 2-dimensional inverse problem on an area-minimizing surface. In particular, we relate our least-area information to knowledge of the Dirichlet-to-Neumann map for the stability operator on a minimal surface.

Broadly speaking, the question we address is a dimension 2 version of the classical boundary rigidity problem for simply connected, Riemannian 3-manifolds with boundary, in which one seeks to determine g given the distance between any two points on the boundary. We will also briefly review this problem of boundary rigidity as it relates to aspects of our question of recovering g from knowledge of areas.

This is joint work with S. Alexakis and A. Nachman.

Calderón problem for Yang-Mills connections

MIHAJLO CEKIĆ (Max Planck Insitute for Mathematics, Bonn)

In this talk, we consider the problem of uniquely identifying a smooth Yang-Mills connection, up to a gauge equivalence fixing the boundary, from the associated Dirichlet-to-Neumann map of the connection Laplacian. We propose two proofs of uniqueness. In the first one we develop a new technique, involving degenerate unique continuation principles and an analysis of the zero set of solutions to an elliptic PDE. The second argument involves a Runge-type approximation along curves to recover holonomy and we are able to show uniqueness of both an arbitrary bundle and a Yang-Mills connection. Although this applies in a more general setting, the first approach uses essentially only one measurement. Time permitting, we will also discuss some (counter) examples concerning zero sets of determinants of matrix solutions to elliptic PDE.

The Poisson embedding approach to the Calderón problem

TONY LIIMATAINEN (University of Helsinki)

We introduce a new approach to the anisotropic Calderón problem, based on a map called Poisson embedding that identifies the points of a Riemannian manifold with distributions on its boundary. We give a new uniqueness result for a large class of Calderón type inverse problems for quasilinear equations in the real analytic case. The approach also leads to a new proof of the result by Lassas and Uhlmann (2001) solving the Calderón problem on real analytic Riemannian manifolds. This is joint work with Matti Lassas and Mikko Salo.

3.2 Tuesday

Inverse problem for Einstein-scalar field equations

LAURI OKSANEN (University College London)

We discuss a method to solve inverse problems for hyperbolic systems where the leading order terms are non-linear. In particular, the method is applied to the coupled Einstein-scalar field equations. The method is based on non-linear interactions of singular solutions to the corresponding linearized equations. The singular solutions are conormal distributions and their non-linear interaction is analysed using the product calculus for conormal distributions. The talk is based on a joint work with Yaroslav Kurylev, Matti Lassas and Gunther Uhlmann.

On distance difference representations of Riemannian manifolds

SERGEI IVANOV (Steklov Institute at St.Petersburg)

I will speak about the following inverse problem introduced by Matti Lassas and Teemu Saksala. Let M be a complete Riemannian manifold, U an open set in M , and F is a “known” open region in M (the “observation domain”). Suppose that for each point x from U one knows the distances from x to all points of F up to an unknown additive constant depending on x . The goal is to determine the topology and metric of U from these data.

As a motivating example, imagine that M is the Earth, the Riemannian metric represents the speed of propagation of elastic waves, F is the Earth’s surface, and U is a seismic active region inside. Suppose that micro-earthquakes occur frequently at points of U at unknown moments of time, and a micro-earthquake occurring at time t at point x produces a wave which arrives to a point y on the surface at time $t + d(x, y)$ where d denotes the Riemannian distance. Then an observer on the surface knows exactly the data described above.

I will show that the geometry of U is indeed uniquely determined by the above data and, in addition, the associated representation of U in a suitable space of functions on F is a locally bi-Lipschitz homeomorphism. This improves earlier results of Lassas and Saksala where it was assumed that $U = M$ and M is compact. The regularity assumptions on the Riemannian metric are only curvature and injectivity radius bounds. This implies stability of the metric determination, via a compactness argument.

Local and global boundary rigidity

PLAMEN STEFANOV (Purdue)

The boundary rigidity problem consist of recovering a Riemannian metric in a domain, up to an isometry, from the distance between boundary points. We show that in dimensions three and higher, knowing the distance near a fixed strictly convex boundary point allows us to reconstruct the metric inside the domain near that point, and that this reconstruction is stable. We also prove semi-global and global results under certain an assumption of the existence of a strictly convex foliation. The problem can be reformulated as a recovery of the metric from the arrival times of waves between boundary points; which is known as travel-time tomography. The interest in this problem is motivated by imaging problems in seismology: to recover the sub-surface structure of the Earth given travel-times from the propagation of seismic waves. In oil exploration, the seismic signals are man-made and the problem is local in nature. In particular, we can recover locally the compressional and the shear wave speeds for the elastic Earth model, given local information. The talk is based on joint work with G.Uhlmann (UW) and A.Vasy (Stanford). We will also present results for a recovery of a Lorentzian metric from red shifts motivated by the problem of observing cosmic strings.

Global analysis for linear and nonlinear waves and the stability of Kerr-de Sitter space

ANDRAS VASY (Stanford)

I will discuss the problem of proving the stability of the family of Kerr-de Sitter (KdS) black holes as solutions of Einstein's vacuum equation: spacetimes evolving from initial data close to those of (M, g) stay globally close to (M, g) , and are indeed asymptotic to (M, g) or another nearby member of the KdS family.

I will focus on analytic aspects of this problem together with the choice of a gauge to break the diffeomorphism invariance of Einstein's equation and the role of constraint damping. The analytic framework is that of global non-elliptic Fredholm problems. The main ingredients are, first, the microlocal control of the regularity of waves by means of elliptic, real principal type, and radial point estimates on a suitable compactification of the spacetime; and second, the asymptotic analysis in which model operators and resonance expansions play a role.

This is joint work with Peter Hintz.

The marked length spectrum of negatively-curved manifolds

THIBAUT LEFEUVRE (Université Paris-Sud)

The marked length spectrum of a negatively-curved manifold is the collection of lengths of closed geodesics, differentiated by their free homotopy classes. It was conjectured by Burns and Katok in the '80s that it should parametrize the set of isometry classes of the manifold — which is not the case of the length spectrum (the collection of lengths, regardless of the homotopy), as proved by Vigneras. In the '90s, Croke and Otal proved independently this conjecture in dimension two but there has not been much progress since and the question remains open. I will present a proof of a local version of the conjecture which holds in any dimension and generalizes to Anosov geodesic flows under some assumptions.

This is a joint work with Colin Guillarmou.

Uniform Sobolev estimates on real hyperbolic spaces

XI CHEN (University of Cambridge)

We shall discuss the uniform resolvent estimates of (p, p') type for $1 \leq p < 2$. As is well-known, the exponents of the classical Sobolev inequality on R^m , due to Kenig-Ruiz-Sogge, are limited to $1 \leq p \leq 2m/(m+2)$, which is related to the Stein-Tomas estimates for the spectral measure. On the other hand, the analogous result on H^{n+1} at high energies turns out to be valid for $1 \leq p < 2$.

Functions of constant X-ray transform

JOONAS ILMAVIRTA (University of Jyväskylä)

Is a non-zero constant function in the range of the X-ray transform in a Euclidean domain or a manifold with boundary? This turns out to be possible on very few domains and manifolds. I will present some phenomena related to this question, including boundary singularities and geometrical restrictions.

This is joint work with Gabriel Paternain.

3.3 Wednesday

Conformal invariance and limiting Carleman weights

DANIEL FARACO HURTADO (Universidad Autonoma de Madrid/ICMAT)

It is well known that for a Riemannian manifold (M, g) admitting a Limiting Carleman Weight is a property of the conformal class. Thus, the presence of LCW implies certain conformal symmetry, part of which can be read from the Cotton and Weyl tensor. Necessary conditions can be obtained which give explicit examples of manifolds not admitting Limiting Carleman Weights. Unfortunately, admitting a LCW imposes additional restrictions on the manifolds and we have found a zoo of example respecting the symmetries imposed by the Weyl and Cotton tensor but not admitting LCW: In fact the more LCW, the more rigid the manifold is and we also discuss how many Limiting Carleman Weight a manifold can admit and what additional symmetries this impose in the manifold. Finally we revisit manifolds where LCW are more abundant, the conformally flat ones, and investigate the structure of LCW in relation with the conformal group. All these are various joint works with P. Angulo, L. Guijarro, A. Ruiz and Mikko Salo.

Eigenvalue bounds for Schrödinger operators with complex potentials and non-trapping metrics

KATYA KRUPCHYK (UC Irvine)

In this talk we shall discuss some recent progress on bounds of Keller and Lieb-Thirring type for eigenvalues of Schrödinger operators with complex potentials on non-trapping asymptotically conic manifolds. While in the self-adjoint case such bounds are classical, the non-self-adjoint case is considerably more involved and the sharpest results currently available were obtained quite recently by R. Frank and collaborators, in the Euclidean setting. We show that such results extend to the general setting of non-trapping asymptotically conic manifolds. A crucial ingredient in our proofs are weighted uniform estimates in suitable Schatten classes for the resolvent of the Laplacian on non-trapping asymptotically conic manifolds. This is joint work with Colin Guillarmou and Andrew Hassell.

Convex projective surfaces with compatible Weyl connection are hyperbolic

GABRIEL PATERNAIN (University of Cambridge)

I will try to give the ideas behind the proof of the following rigidity result: a properly convex projective structure on a closed oriented surface of negative Euler characteristic arises from a Weyl connection if and only if it is hyperbolic. We phrase the problem as a non-linear PDE for a suitable Beltrami differential. Turning this non-linear PDE into a transport equation, we obtain the result by applying methods from geometric inverse problems; more specifically, tensor tomography for tensors of order 3.

This is joint work with Thomas Mettler.

Unique recovery of faults and dislocations from coseismic displacement data

MAARTEN V. DE HOOP (Rice University)

An elastic dislocation is an internal open and oriented surface in an elastic solid across which there is a discontinuity of the displacement. It describes a fault plane undergoing slip over a limited area, a thin intrusion or a crack whose faces slide over one another or separate by the action of an applied stress. An elastic dislocation where the displacement discontinuity varies from point to point of the internal surface is called Somigliana dislocation while the particular case of a constant displacement discontinuity has been referred to as Volterra dislocation. Assuming the earth isotropic and a nonhomogeneous infinite medium, in the regime of small-amplitude deformations, we are led to the study of a boundary value problem in a half-space for the system of linearized elasticity. We prove, by using unique continuation properties of solutions to the Lamé system with Lipschitz Lamé parameters that one surface measurement of the displacement field is sufficient to recover uniquely both the fault and the slip, assuming that the fault is piecewise linear and the slip field purely tangential directed in the normal direction. In the context of earthquakes and geodesy, the problem comprises coseismic deformation with GPS or InSAR data.

Joint work with A. Aspri, E. Beretta and A. Mazzucato.

Inverse problems with one measurement

EEMELI BLÅSTEN (HKUST Jockey Club Institute for Advanced Study)

Inverse scattering and boundary value problems were traditionally solved by an infinite number of measurements, as done by Sylvester and Uhlmann. It is also well known that a single measurement is not always enough. However under quite general conditions it is possible to find useful information about the unknown from a single measurement. Such conditions include for example the scatterer having a polyhedral shape. This topic also has wide-ranging implications: to invisibility, the interior transmission problem, non-scattering sources and the inverse source problem.

Uniqueness of a potential from boundary data in locally conformally transversally anisotropic geometries

ALI FEIZMOHAMMADI (University College London)

Let (Ω^3, g) be a compact smooth Riemannian manifold with smooth boundary and suppose that U is an open set in Ω such that $g|_U$ is the Euclidean metric. Let $\Gamma = \bar{U} \cap \partial\Omega$ be connected and suppose that U is the convex hull of Γ . We will study the uniqueness of an unknown potential for the Schrödinger operator $-\Delta_g + q$ from the associated Dirichlet to Neumann map, Λ_q . We will prove that if the potential q is a priori explicitly known in U^c , then one can uniquely reconstruct q over the convex hull of Γ from Λ_q . We will also outline a reconstruction algorithm. More generally we will discuss the cases where Γ is not connected or $g|_U$ is conformally transversally anisotropic and derive the analogous result. In the final section of the paper, we will study the question of determining the potential from the knowledge of the local Dirichlet to Neumann map, $C_q^{\Gamma, \Gamma}$.

On localizing and concentrating electromagnetic fields

YI-HSUAN LIN (Institute for Advanced Study, HKUST)

We consider field localizing and concentration of electromagnetic waves governed by the time-harmonic anisotropic Maxwell system in a bounded domain. It is shown that there always exist certain boundary inputs which can generate electromagnetic fields with energy localized/concentrated in a given subdomain while nearly vanishing in another given subdomain. The theoretical results may have potential applications in telecommunication, inductive charging and medical therapy. We also derive a related Runge approximation result for the time-harmonic anisotropic Maxwell system with partial boundary data.

3.4 Thursday

Plasmon resonances in optics and elasticity and their applications

HONGYU LIU (Hong Kong Baptist University)

I shall talk about our recent progress on the mathematical study of plasmon resonances and their applications. Plasmon materials are materials with negative parameters and can induce various resonance phenomena. In our study, we combine the spectral approach and the variational approach. In particular, we discover some interesting spectral properties of the Neumann-Poincare operators during the process.

Attenuated tensor tomography on the Euclidean disk and the inverse source problem in transport

FRANÇOIS MONARD (University of California Santa Cruz)

Over the past decade, the tensor tomography problem *what can be reconstructed of a tensor field on a Riemannian manifold from its integrals along geodesics and how to reconstruct it* has received a new wave of positive answers to the typical inverse problems questions. On the other hand, some arguments in a general Riemannian context, based on energy identities and microlocal methods, remain to be upgraded to constructive and implementable answers.

In this talk, we will give a tour of the case of the Euclidean unit disk, where several answers can be made explicit, including:

- (1) reconciling range characterizations associated with parallel and fan-beam geometries (Helgason-Ludwig conditions vs Pestov-Uhlmann range characterization), a Euclidean-specific issue,
- (2) how to reconstruct gauge representatives for tensor fields,
- (3) how to construct special invariant distributions for the previous purpose, and what is their optimal regularity,
- (4) how to adjust these results to the case of the attenuated X-ray transform.

Time allowing, we will explain how these results also help solving the inverse source problem in Boltzmann transport when the scattering kernel has finite harmonic content, a problem with applications to Optical Molecular Imaging.

Microlocal analysis of Doppler synthetic aperture radar

ALLAN GREENLEAF (University of Rochester, USA)

Standard synthetic aperture radar (SAR) images terrain or objects on the ground using high resolution range data obtained from measurements of scattered EM waves. A transceiver on a moving aircraft or satellite emits short pulse (hence broadband) waves and then measures the scattered waves which reflect off of the target. In contrast, we study a system using a narrow band (hence long duration) wave to produce high resolution Doppler data; we refer to this as Doppler-based SAR (DSAR). A windowed Fourier transform converts the raw data into a function of two variables: slow time along the flight path and frequency. Under simplifying assumptions, we show that the linearized forward scattering map is a Fourier integral operator and study the feasibility of inverting it via filtered backprojection, focusing on artifacts and their possible avoidance or suppression. Initially derived under a start-stop approximation widely used in range-based SAR, we show that some of our results for DSAR are robust and hold under a more realistic approximation.

This is joint work with Raluca Felea, Romina Gaburro and Cliff Nolan.

Geometric inverse problems with random and unknown sources

MATTI LASSAS (University of Helsinki)

We consider the determination of the Riemannian metric on a compact or complete Riemannian manifold (M, g) when we observe waves that produced by sources that we can not control. Such random or poorly known sources are often encountered in passive imaging. For example, we consider the wave equation $(\partial_t^2 - \Delta_g)u(x, t) = f(x, t)$ where $f(x, t)$ is a white noise supported in domain $W \times \mathbb{R}_+$ and we observe the waves in the set $V \times \mathbb{R}_+$, and show that under appropriate geometric assumptions the metric can be uniquely determined when $V \cap W \neq \emptyset$.

We also consider the determination of metric g when we observe waves $u(x, t)$ produced by several sources f that are point sources located in unknown points. The results involving the white noise source are done in collaboration with Tapio Helin, Lauri Oksanen and Teemu Saksala.

4 Poster abstracts

Decay estimates in evolution equations with fractional time derivative

ELISA AFFILI (Università degli Studi di Milano)

We consider an evolution equation with possibly fractional diffusion. The anomalous diffusion may occur either in space or in time (or both) and the diffusion operator can be also nonlinear. We prove some power-law decay estimates in time, finding a polynomial decay that depends on the parameter of the fractional time derivative. Also, in the case of evolution equation involving only classical time derivative, we are able to improve the power-law decay to an exponential one. We are able to prove the decays in a general setting by using a structural hypothesis; then, it is important to point out that many operators satisfy it. Among these there are the Laplacian, the p -Laplacian, the Laplacian, the porous medium equations, the magnetic operator and their fractional counterparts. In fact, the hypothesis holds essentially when it is possible to perform an integration by parts of the energy functional. We believe that the result is very general and applies to a very broad class of operators. This work was done in collaboration with Enrico Valdinoci.

Inverse problem for fractional Laplacian with non-local lower order perturbation

SOMBUDDHA BHATTACHARYYA (Institute for Advanced Study, The Hong Kong University of Science and Technology)

We consider a non-local inverse problem and determine more than one lower order coefficients from the associate Cauchy data. Apart from the global non-locality in the principal part, our operator exhibits regional non-locality in its lower order perturbation.

Variable exponent Calderón's problem in one dimension

TOMMI BRANDER (Technical University of Denmark)

We consider Calderón's problem for variable exponent $p(x)$ -Laplace equation and show that, in one dimension, the problem is more interesting than constant exponent Calderón's problem. We show a uniqueness result for conductivities that are measurable with respect to the coarsest sigma-algebra which makes the variable exponent $p(x)$ measurable. The value of the conductivity can be explicitly recovered at the minimum and maximum points of $p(x)$, under some assumptions. Also, the analogue of the total resistivity can be recovered, as with constant exponent Calderón's problem.

Inverse problems for a fractional conductivity equation

GIOVANNI COVI (University of Jyväskylä)

We show global uniqueness in two inverse problems for a fractional conductivity equation: an unknown conductivity in a bounded domain is uniquely determined by measurements of solutions taken in arbitrary open, possibly disjoint subsets of the exterior. The results are based on a reduction from the fractional conductivity equation to the fractional Schrödinger equation.

Inversion of a restricted transverse ray transform on symmetric tensor fields in Euclidean space

SUMAN KUMAR SAHOO (TIFR-CAM Bangalore)

We consider microlocal inversion of a restricted transverse ray transform on symmetric m -tensor fields in $\mathbb{R}^n$ for $n \geq 3$. More precisely, we show that a symmetric m -tensor field can be recovered modulo a known singular error term and smoothing terms if its transverse ray transform is known along all lines intersecting a given smooth curve satisfying certain conditions.

Reconstruction of piecewise constant functions from X-ray data

VADIM LEOVICI (ENS Paris)

We show that on a two-dimensional compact nontrapping Riemannian manifold with strictly convex boundary, a piecewise constant function can be recovered from its integrals over geodesics. We adapt the injectivity proof using variations through geodesics and we improve this result when the manifold is simple and the function is constant on tiles with geodesic edges, showing that the Jacobi fields of these variations are sufficient. We give also explicit formulas for the values near the boundary. We finally study the stability of the reconstruction method.

Global identifiability of low regularity fluid parameters in acoustic tomography of moving fluid

BOYA LIU (University of California, Irvine)

We discuss inverse boundary problems for first order perturbations of the Laplacian, which arise as model operators in the acoustic tomography of a moving fluid. We show that the knowledge of the Dirichlet-to-Neumann map on the boundary of a bounded domain in $\mathbb{R}^n$, $n \geq 3$, determines the first order perturbation of low regularity up to a natural gauge transformation, which sometimes is trivial. As an application, we recover the fluid parameters of low regularity from boundary measurements, sharpening the regularity assumptions in the recent results of Agaltsov and Novikov. In particular, we allow some fluid parameters to be discontinuous.

A resolvent estimate for the magnetic Schrödinger operator in the presence of short and long-range potentials

CRISTÓBAL J. MEROÑO[†], LEYTER POTENCIANO-MACHADO[‡], MIKKO SALO[‡] (Universidad Autónoma de Madrid[†], University of Jyväskylä[‡])

It is well known that the resolvent of the free Schrödinger operator on weighted L^2 spaces has norm decaying like $\lambda^{-1/2}$ at energy λ . Combining multiplier techniques, a positive commutator method, Carleman estimates and integration by parts, we show that this result is valid for perturbations of the free case by magnetic and electric potentials satisfying short and long-range conditions at infinity.

Keywords: Schrödinger operator, short and long-range potentials, Carleman estimates, Agmon spaces.

Artifacts in the inversion of the broken ray transform in the plane

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We study the integral transform over a general family of broken rays in $\mathbb{R}^2$. There is a natural notion of conjugate points for broken rays. If there are conjugate points, we show that the singularities conormal to the broken rays cannot be recovered from local data and therefore artifacts arise in the reconstruction. As for global data, more singularities might be recoverable. We apply these conclusions to two examples, the V-line transform and the parallel ray transform. In each example, a detailed discussion of the local and global recovery of singularities is given and we perform numerical experiments to illustrate the results.

Some Sobolev regularity for ∞ -Laplace equation in the plane

YI ZHANG (Mathematical Institute of the University of Bonn)

Infinity Laplace equations are highly degenerated nonlinear elliptic equations. In our papers [1, 2] we show mainly the $W_{\text{loc}}^{1,2}$ -regularity of $|Du|^2$ for (both homogeneous and nonhomogeneous) infinity Laplace equations in the plane, together with some sharp regularity of other terms.

[1] H. Koch, Y. R.-Y. Zhang, Y. Zhou, *An asymptotic sharp Sobolev regularity for planar infinity harmonic functions*, arXiv:1806.01982.

[2] H. Koch, Y. R.-Y. Zhang, Y. Zhou, *Some sharp Sobolev regularity for inhomogeneous ∞ -Laplace equation in plane*, arXiv:1806.01987.