



Applications:

1) Calderon problem in  $\Omega \subset \mathbb{R}^n$  bounded  $C^\infty$  domain:

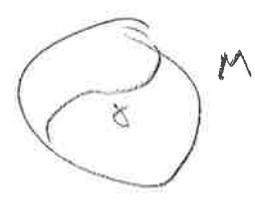
$$\begin{cases} \operatorname{div}(\kappa(x)\nabla u) = 0 & \text{in } \Omega \\ u = f & \text{on } \partial\Omega \end{cases}$$

DN map  $\Delta_\kappa : f \mapsto \partial_\nu u|_{\partial\Omega}$  is in  $\mathcal{E}^1(\partial\Omega)$ .

Principal symbol of  $\Delta_\kappa$  det.  $\kappa|_{\partial\Omega}$ , sub-principal symbol det.  $\partial_\nu \kappa|_{\partial\Omega}$ , ..., and full symbol of  $\Delta_\kappa$  (in certain coordinates) det. Taylor series of  $\kappa$  on  $\partial\Omega$ .

2) Geodesic X-ray transform in compact mfld  $(M, g)$  with boundary: if  $f \in C^\infty(M)$

$$If(x) = \int_\gamma f(x(t)) dt.$$



Under certain assumptions on  $(M, g)$ ,  $I$  is a FIO of order  $-1/2$  and  $I^*I$  is an elliptic  $\mathcal{E}DO$  of order  $-1$  on  $M$ . Microlocal analysis:

if  $If = 0$ , then  $f \in C^\infty(M)$  and thus main singularities of  $f$  can be recovered from  $If$ .

3) Gauss-Bonnet theorem: if  $(M, g)$  is a closed oriented 2d mfld, then

$$\frac{1}{2\pi} \int_M K dV = \chi(M) \quad \left( \begin{array}{l} \text{special case of} \\ \text{Atiyah-Singer index theorem} \end{array} \right)$$

Here  $\chi(M)$  is the Fredholm index of  $A = d + d^* : \Lambda^{\text{even}} \rightarrow \Lambda^{\text{odd}}$  ( $\operatorname{Ind}(A) = \dim \operatorname{Ker}(A) - \dim \operatorname{Ker}(A^*) = b_0(M) - b_1(M) + b_2(M)$ ), and  $\frac{1}{2\pi} \int_M K dV$  is  $\operatorname{ind}(A)$  computed in another way.

4) Eigenvalue asymptotics: let  $\Omega \subset \mathbb{R}^n$  be a bounded domain, let  $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots \rightarrow \infty$  be Dirichlet e.w.s of  $-\Delta$  in  $\Omega$ :

$$\begin{cases} -\Delta \phi_j = \lambda_j \phi_j & \text{in } \Omega \\ \phi_j|_{\partial\Omega} = 0 \end{cases}$$

and let  $N(\lambda) = \#\{\lambda_j ; \lambda_j \leq \lambda\}$ . Then

$$N(\lambda) \sim \frac{|\Omega|}{\Gamma(\frac{n+2}{2})(4\pi)^{n/2}} \lambda^{n/2} \quad \text{as } \lambda \rightarrow \infty.$$

Proof: estimate heat kernel  $\operatorname{Tr}(e^{-t\Delta}) = \sum_{j=0}^\infty e^{-t\lambda_j}$  as  $t \rightarrow 0$  by microlocal methods, and use a Tauberian theorem.