

## Analysis on manifolds

Possible topics for seminars/essays  
(first week of December, decide by Thu 06.11.)

1. Hodge theory [Ta, Section 5.8]:
  - Gaffney's inequality [Ta, Proposition 5.8.1]
  - Elliptic regularity for the Hodge Laplacian [Ta, Theorem 5.1.3]
2. Morse theory [Mi, Part 1], [MT, Section 12]:
  - Morse coordinates [MT, Theorem 12.6]
  - Existence of Morse functions [MT, Theorem 12.4]
3. Conformal and quasiconformal maps on Riemannian manifolds:
  - Liouville's theorem in  $\mathbb{R}^n$ ,  $n \geq 3$  [IM, Chapter 2]
  - Isothermal coordinates on Riemann surfaces [Ta, Section 5.10]
  - Uniformization theorem [Ta, Section 14.2]
4. More about geodesics:
  - Geodesics minimize locally [Le97, Theorem 6.12]
  - Second variation formula [Le97, Theorem 10.12]
  - Conjugate points [Le97, Theorem 10.15]
5. Inverse problems on manifolds (geodesic ray transform)
6. Lower bounds for Ricci curvature
7. Ricci flow and Perelman's solution of the Poincaré conjecture

## References

- [IM] T. Iwaniec, G. Martin, *Geometric function theory and non-linear analysis*, Clarendon Press, Oxford, 2001.
- [MT] I. Madsen, J. Tornehave, *From calculus to cohomology*. Cambridge University Press, 1997.
- [Mi] J. Milnor, *Morse theory*. Princeton University Press, 1963.
- [Le97] J.M. Lee, *Riemannian manifolds*. An introduction to curvature. Springer, 1997.
- [Ta] M.E. Taylor, *Partial differential equations I-III*. Springer, 1996.