Analysis on manifolds
Questions #1, 01.10.2014
(discussion Wed 08.10.2014)

1. Can the curl operator be generalized to n dimensions?
2. In $dx^{j_1} \wedge \ldots \wedge dx^{j_k} = (-1)^{\mathrm{sgn}(\sigma)} dx^{j_{\sigma(1)}} \wedge \ldots \wedge dx^{j_{\sigma(k)}}$, can we use the Levi-Civita symbol instead of $(-1)^{\mathrm{sgn}(\sigma)}$?
3. The gradient can be thought of as a covariant derivative in general; does the gradient increase the order of the tensor by one?
4. Can we talk about a decomposition of vector fields in $L^2(U, \mathbf{R}^3)$ into a curl-free and divergence-free part?
5. What do the gradient, curl and divergence mean geometrically?
6. What does the Laplacian mean geometrically?
7. What kinds of spaces can a Laplace operator be defined in?
8. Is there an analytic way to see why the de Rham cohomology groups are topological invariants?