Analysis on manifolds
Questions #2, 16.10.2014
(discussion Wed 22.10.2014)

1. If $\gamma : [a, b] \to U$ is a continuous function, the length of γ can also be defined by $\tilde{L}_g(\gamma) = \sup_{a=t_0<\ldots<t_{N+1}=b} \sum_{j=1}^{N+1} d_g(\gamma(t_j), \gamma(t_{j-1}))$ where the supremum is taken over all subdivisions (the curve is called rectifiable if $\tilde{L}_g(\gamma) < \infty$). Is this compatible with the definition given in the lectures?
2. Given a smooth manifold M, by the Whitney embedding theorem we may embed M in $\mathbb{R}^N$ for some N. Does M become a Riemannian manifold?
3. Is it possible to compute the first variation of the length functional as a Fréchet derivative in some infinite-dimensional space?
4. What can be expected about the second variation of the length functional, in particular when evaluated at a length minimizing curve?
5. Are there any natural conditions ensuring that a solution of the geodesic equation minimizes the distance between its endpoints?
6. Is the codifferential operator δ related to some operator in singular homology or cohomology?