Inverse Days 2012, Jyväskylä

List of Abstracts

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Harri Auvinen (University of Helsinki)

Utilizing Markov Chain Monte Carlo (MCMC) Method for Improved Glottal Inverse Filtering

Abstract: Glottal inverse filtering (GIF) is a technique for estimating the glottal volume velocity waveform (i.e. the glottal source) from a speech signal. Glottal inverse filtering involves first estimating the vocal tract filter, which is then used to cancel the effect of the vocal tract by filtering the signal by the inverse model of the tract. The resulting signal is the time-domain waveform of the estimated glottal source. Glottal inverse filtering is an inverse problem that is difficult to solve accurately. Moreover, it is not possible to observe non-invasively the glottal source, and thus it is difficult to validate the accuracy of GIF.

In our approach, computational inversion methods are utilized in order to improve the accuracy of GIF. More specifically, we take advantage of Markov chain Monte Carlo (MCMC) methods in order to determine new vocal tract models for GIF. The method is based on first finding an initial estimate of the vocal tract filter using the iterative adaptive inverse filtering (IAIF), and then refining the GIF model parameters within the MCMC method in order to get optimal inverse filtering result. The goodness of the results is evaluated by comparing the original speech signal to a synthetic one created by filtering the Klatt model based excitation signal with the estimated vocal tract filter. The GIF model parameters include a few first poles of the vocal tract filter and the Klatt parameter. Experiments, demonstrated at the conference, show that the MCMC-based GIF method improves the accuracy of the results more than 10% at the higher fundamental frequency of the vowels. Furthermore we introduce a concept of parallel glottal inverse filtering using Techila high-performance computing (HPC) environment.

Francis Chung (University of Jyväskylä)

Partial Data for the Neumann-Dirichlet Map

Abstract: I will show that an electric potential can be uniquely determined from partial knowledge of the Neumann-Dirichlet map for the corresponding Schrödinger operator. The result is analogous to the one by Kenig, Sjöstrand, and Uhlmann for the Dirichlet-Neumann map. The main new ingredient is a Carleman estimate for the Schrödinger operator with appropriate boundary conditions.

Matias Dahl (Aalto University)

Geometrisation of Gaussian beams in the scalar wave equation

Abstract: *Gaussian beams* are a class of asymptotic solutions to hyperbolic equations that behave as wave packets; a Gaussian beam propagates along a curve in space, and at each time instant the energy of the Gaussian beam is concentrated around one point along the curve. A key feature for Gaussian beams is that their propagation is determined by ordinary differential equations along the underlying curve. This implies that one can determine the properties of a Gaussian beam without solving the full wave equation. Due to this feature Gaussian beams have been used both as a tool for theoretical questions, but also as a tool for practical problems.

We will study the scalar wave equation on a Riemannian manifold. In this setting, the construction of Gaussian beams is based on Taylor expansions of the phase- and amplitude functions in local coordinates on the manifold. It is known that the first few coefficients in these expansions can be rewritten as solutions to tensor equations; a Gaussian beam propagates along a geodesic, its shape is determined by a complex tensorial Ricatti equation and the leading amplitude is determined a complex Jacobi equation. In this talk we address the task of a geometric construction for Gaussian beams that avoids the use of Taylor expansions in local coordinates and where all objects are tensorial.

Josef Durech (Charles University in Prague)

Asteroids@home - asteroid lightcurve inversion with distributed computing

Abstract: I will present a new project called Asteroids@home (http://asteroidsathome.net). It is a volunteer-computing project that uses an open-source BOINC (Berkeley Open Infrastructure for Network Computing) software to distribute tasks to volunteers, who provide their computing resources. The project was created at the Astronomical Institute, Charles University in Prague, in cooperation with the Czech National Team. The scientific aim of the project is to solve a time-consuming inverse problem of shape reconstruction of asteroids from sparse-in-time photometry. The time-demanding nature of the problem comes from the fact that with sparse-in-time photometry the rotation period of an asteroid is not apriori known and a huge parameter space must be densely scanned for the best solution. The nature of the problem makes it an ideal task to be solved by distributed computing - the period parameter space can be divided into small bins that can be scanned separately and then joined together to give the globally best solution. In the framework of the the project, we process asteroid photometric data from surveys and we derive asteroid shapes and spin states. The algorithm is based on the lightcurve inversion method developed by Kaasalainen et al. (Icarus 153, 37, 2001). The enormous potential of distributed computing will enable us to effectively process also the data from future surveys. We also plan to process data of a synthetic asteroid population to reveal biases of the method.

Daniel Gerth (Johannes Kepler University Linz)

Stochastic convergence analysis for Tikhonov-Regularization with sparsity constraints

Abstract: This is joint work with Ronny Ramlau. In recent years, regularization methods based on the minimization of Tikhonov-type functionals

$$||Ax - y||^2 + \alpha\Phi(x) \tag{1}$$

with a linear bounded operator A and a sparsity promoting penalty term Φ have been discussed widely in literature (see for example [3]). Convergence of the solution has been analysed assuming a deterministic error bound

$$||y - y^\delta|| \leq \delta \tag{2}$$

between the measured data y^δ and the true data y. Instead of ((2)), an explicit stochastic error model is considered in the talk. Namely, the case of a normally distributed error with zero mean and variance σ^2 in each component of the measured data is considered. This especially means arbitrarily large errors are allowed, but with low probability. Equation ((1)) is derived from a stochastic model using Bayes' formula (cf e.g. [1]). Deterministic results are lifted to this situation using the Ky Fan metric for the convergence analysis (see for example [2]). After giving a general convergence theorem, Besov space penalties are considered (see, e.g. [3], [4]). For this case, a parameter choice rule is presented which immediately leads to convergence rates in the Ky Fan metric with respect to the error parameter σ.

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Drossos Gintides (National Technical University of Athens)

The inverse transmission eigenvalue problem for spherically symmetric refractive index

Abstract: In this talk we will present results concerning the recovery of a spherically symmetric function which describes the index of refraction for Helmholtz equations or a potential for Schrödinger equations from the set of the corresponding transmission eigenvalues with spherically symmetric eigenfunctions. We will show that under specific conditions these functions are uniquely determined by the data consisting of such transmission eigenvalues and their multiplicities. We will also present numerical results for a simple spherically symmetric refractive index using the knowledge of few transmission eigenvalues. The reconstructions can be done using a weak formulation of the problem and considering the corresponding inverse discrete quadratic problem.

Roland Griesmaier (Leipzig University)

Far field splitting for the Helmholtz equation

Abstract: We consider the inverse source problem for the two-dimensional Helmholtz equation as a means to provide an inversion algorithm for inverse scattering problems for acoustic or electromagnetic waves with a single excitation. Probing an ensemble of obstacles by just one primary wave at a fixed frequency and measuring the far field pattern of the corresponding scattered wave, the inverse scattering problem that we are interested in consists in reconstructing the support of the scatterers. To this end we rewrite the scattering problem as a source problem and apply two recently developed algorithmsthe inverse Radon approximation and the convex scattering supportto recover information on the support of the corresponding source. The first method builds upon a windowed Fourier transform of the far field data, followed by a filtered backprojection. Although the inverse Radon approximation yields a rather blurry reconstruction, it can be applied to identify the number and the positions of well separated source components. This information is then utilized to split the far field into individual far field patterns radiated by each of the well separated source components, and to compute the associated convex scattering supports as a reconstruction of the individual scatterers. We discuss this algorithm and consider numerical results.

Sarah Hamilton (University of Helsinki)

Direct D-bar reconstructions of complex admittivities on a chest-shaped domain in 2D EIT

Abstract: An implementation of a non-iterative D-bar method for complex admittivities for the Calderón problem on a general 2D domain is presented. The framework is based on the uniqueness proof by Francini [Inverse Problems **20** 2000] and subsequent complete D-bar algorithm by Hamilton et al. [Inverse Problems **28** 2012]. The algorithm is parallelizable and employs a low frequency limit in the scattering parameter. Key tools in the method include complex geometrical optics solutions, scattering transforms, and D-bar equations. Electrical impedance tomography reconstructions from numerically simulated noisy data are computed on a chest-shaped domain for several realistic 2D phantoms including a simulated pneumothorax, hyperinflation, and pleural effusion with discontinuities at the organ boundaries.

Markus Harju (University of Oulu)

A solution method and numerical examples for TE waves

Abstract: This is joint work with V. Serov and H. W. Schürmann. We consider the propagation of electromagnetic waves in an anisotropic medium in $\mathbb{R}^3$. We investigate a three-layer structure with film, cladding and substrate. The permittivity is assumed to be constant outside film and of very general nonlinear type in the film. Moreover, we assume that there is no electric field in the direction of propagation, the so called TE case. We present a method for the solution of electric and magnetic fields. The dispersion relation is also discussed with numerical examples for a Kerr-type (cubic) nonlinearity.

Joonas Ilmavirta (University of Jyväskylä)

Broken ray tomography in the disk

Abstract: We introduce the broken ray transform in a Euclidean domain. Injectivity of this transform is related to partial data problems and is of interest in its own right. We study the problem in a Euclidean ball when measurements can be made in a singleton or a small open subset of the boundary. With suitable regularity assumptions the broken ray transform is injective for an open set for measurements, but for a singleton the reconstruction is only possible at the origin.

Kim Knudsen (Technical University of Denmark)

3D inversion in EIT by regularized scattering transform

Abstract: The Calderón problem is the mathematical formulation of Electrical Impedance Tomography. In 3D the problem was solved analytically in the mid 1980's, but only recently a numerical implementation of the reconstruction algorithm was made. In this talk the algorithm and its complete implementation and regularization will be presented. We will discuss the benefits and challenges of the method, and evaluate the performance of the algorithm on several numerical examples.

This is joint work with Fabrice Delbary and Per Christian Hansen, Technical University of Denmark.

Janne Koponen (University of Eastern Finland)

Approximation Error Method for Full-Wave Tomography

Abstract: This is joint work with Tomi Huttunen, Tanja Tarvainen, and Jari P. Kaipio. In ultrasound tomography (UT), the speed of sound (SOS) is reconstructed based on ultrasound measurements made on the surface of the object. As a part of the reconstruction process propagation of acoustic signals in the medium is simulated using a forward model. Consequently, modeling errors can generate artifacts into reconstructed SOS. Accurate full-wave models can be computationally heavy, and thus impractical in many real applications. On the other hand, approximate models typically lead to less accurate reconstructions. In this study, measurement noise and modeling errors of UT are modeled in Bayesian framework, and a numerical method that takes the approximation errors into account is developed. The performance of the method is investigated by numerical simulations in which artifacts generated by a fast but less accurate forward model and approximate boundary conditions are compensated.

Marko Järvenpää (Tampere University of Technology)

Bayesian Hierarchical Modelling for Image Deblurring

Abstract: A Bayesian hierarchical model allows all the parameters of an inverse problem, including the "regularisation parameter", to be estimated simultaneously from the data. In this work a Bayesian hierarchical model for total variation regularisation is presented. The model is based on the characterisation of the Laplace density prior as a scale mixture of Gaussians. With different priors on the mixture variable, other total variation like regularisations are obtained. An approximation of the posterior mean is found using a variational bayes method. The approach is illustrated with examples of image deblurring and image denoising.

Marko Laine (Finnish Meteorological Institute)

Parameter estimation in large scale models using ensemble runs

Abstract: I will talk about tuning of numerical weather prediction and climate models. Model tuning is a short name for a variety of approaches in inverse problems of non-linear parameter estimation in geophysical systems. In multi scale dynamical state space models, sub-grid scale processes must be replaced by physical parameterization schemes. Tuning and uncertainty analysis of closure parameters related to these schemes is complicated by the high complexity and computational burden of the models. As the number of times the model can be evaluated sequentially is limited, we need to find methodologies that utilize new parallel computer infrastructures. Also, in state space models, the uncertainty in the initial values of the system state has to be integrated out for the model closure parameter estimation. Ensemble methods provide solution to both parallelization and initial value uncertainty. The third source of uncertainty coming from modelling error can be handled by suitable hierarchical statistical description of the parameter uncertainties. In this talk, I will describe a new model ensemble based parameter estimation system, experiments with Lorenz 95 benchmark model, with ECHAM5 climate model and the implementation of the system in European Centre for Medium-Range Weather Forecasts (ECMWF) for their numerical weather prediction ensemble framework.

Matti Lassas (University of Helsinki)

Reconstruction of wave speed in a seismic inverse problem

Abstract: We analyze the inverse problem, originally formulated by Dix in geophysics, of reconstructing the wave speed inside a domain from boundary measurements associated with the single scattering of seismic waves. We consider a domain M with a varying and possibly anisotropic wave speed which we model as a Riemannian metric g. For our data, we assume that M contains a dense set of point scatterers and that in a subset $U \subset M$, modeling the domain that contains the measurement devices, e.g, on the Earth's surface is seismic measurements, we can produce sources and measure the wave fronts of the single scattered waves diffracted from the point scatterers. The inverse problem we study is to recover the metric g in M up to a change of coordinates. To do this we show that the shape operators related to wave fronts produced by the point scatterers within M satisfy a certain system of differential equations which may be solved along geodesics of the metric. In this way, assuming we know g as well as the shape operator of the wave fronts in the region U, we may recover g in certain coordinate systems (i.e. Riemannian normal coordinates centered at point scatterers). This generalizes the well-known geophysical method of Dix to metrics which may depend on all spatial variables and be anisotropic. In particular, the novelty of this solution lies in the fact that it can be used to reconstruct the metric also in the presence of the caustics.

The results have been done in collaboration with Maarten de Hoop, Sean Holman, Einar Iversen, and Bjorn Ursin.

Felix Lucka (University of Münster)

Sparsity Constraints in Bayesian Inversion

Abstract: Sparsity has become a key concept for solving of high-dimensional ill-posed inverse problems. This talks focuses on two ways of using sparsity constraints in the framework of Bayesian statistics:
(1) Turning the functionals used in variational regularization into prior distributions in the Bayesian setting and analyzing the properties of the resulting Bayesian estimators. A practical obstacle for this is the lack of fast posterior sampling algorithms. In the talk, a new implementation of a single component Gibbs MCMC sampler is developed and examined. It is demonstrated that in contrast to Metropolis Hastings MCMC schemes, its efficiency increases when the impact of the sparsity constraints or the dimension of the unknowns is increased. These results also challenge common beliefs about the applicability of sample based Bayesian inference for high dimensional inverse problems in general.
(2) Using hierarchical Bayesian modeling (HBM) with heavy-tailed hyperpriors. This is a current trend in all areas of inverse problems. We show that for the severely ill-posed inverse problem of EEG/MEG source reconstruction, HBM-based inference shows very promising results for the recovery of brain networks involving deep-lying sources.

Teemu Luostari (University of Eastern Finland)

Modeling of thin clamped plates using an efficient non-polynomial method

Abstract: This is joint work with Tomi Huttunen and Peter Monk. Numerical modeling of thin plates arises in structural dynamics. A fourth order partial differential equation, based on Kirchhoff's plate theory, needs to be solved in order to find the displacement field. Traditionally, at lower frequencies, the finite element method (FEM) is used to solve the plate bending problems. However, at higher frequencies the computational burden of the FEM becomes too large and commonly more efficient non-polynomial methods are then used. Generally speaking, during the past 15 years, the non-polynomial methods have become more and more popular. The ultra weak variational formulation (UWVF) is a non-polynomial method, belongs to the class of Trefftz methods and uses finite element meshes. When modeling the plate dynamics, the plane and evanescent (corner) wave basis functions are used in the UWVF. The advantage of aforementioned physical basis functions is the fact that the integrals in the sesquilinear form can be computed in closed form. We shall focus on Kirchhoff's thin plate theory and investigate numerically the applicability of the UWVF in thin clamped plate problems.

Anu Määttä (Finnish Meteorological Institute)

Quantification of model uncertainty in satellite aerosol optical thickness retrieval

Abstract: This talk is about uncertainty quantification in aerosol optical thickness (AOT) retrieval based on satellite measurements. The inversion is made by using multidimensional look-up tables containing a number of microphysical aerosol models as a result of radiative transfer calculations. The uncertainty in the retrieval consists of measurement noise, uncertainty in aerosol model choice, and uncertainty in aerosol models. The amount of information in satellite measurement may not be enough for selecting the correct aerosol model, and there can be several alternative models that fit to observations equally well. Furthermore, the models are only simplifications of the real aerosol situation in the atmosphere. We have modeled this discrepancy between aerosol model and the observed state using Gaussian processes. In addition to the measurement noise, the model discrepancy term allows for smooth deviations of the modelled values from the observed ones. Finally, the retrievals from the most appropriate aerosol models are averaged by weighting each model by its relative evidence. By these methods we can effectively account for modelling uncertainties.

Dinh-Liem Nguyen (École Polytechnique)

On Shape Identification of Diffraction Gratings from Spectral Data

Abstract: This is joint work with Armin Lechleiter. We consider the shape identification problem of diffraction gratings from measured spectral data involving of scattered electromagnetic waves. The model problem that we study here is motivated by the important applications of such structures in op- tics. Applications include diffractive optical filters and organic light-emitting diodes, and non-destructive testing is an important issue to guarantee the functioning of such devices. Recently, the Factorization method, introduced in [3], has been extended to periodic inverse scattering problems as a tool for imaging. In [1, 2] the authors studied the Factorization method for the imaging problem of impenetrable periodic structures with Dirichlet and impedance boundary conditions. The paper [4] considered imaging of penetrable periodic interfaces between two dielectrics in two dimensions. In the present work, the Factorization method has been studied for identifying shape of diffraction gratings constituted by penetrable periodic/biperiodic dielectrics in three dimensions. We provide a rigorous analysis for the method as well as numerical experiments to examine its performance.

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Aki Pulkkinen (University of Eastern Finland)

Truncated Fourier series approximation of the time-domain radiative transfer equation using finite elements

Abstract: This is joint work with Tanja Tarvainen. Radiative transfer equation is a widely used model for the propagation of light in scattering media, such as soft-tissues. We have proposed a method for approximating the solution of the time-domain radiative transfer equation using truncated Fourier series. In this study we compare the proposed method to direct numerical integration methods of the radiative transfer equation. In addition we compare the computations to time domain Monte Carlo simulations.

Juan Reyes (University of Helsinki)

Otto Seiskari (Aalto University)

Calderón's problem with partial data in piecewise smooth plane domains

Abstract: Conductivity equation is studied in piecewise smooth plane domains and with measure-valued current patterns (Neumann boundary values). This allows one to extend the recently introduced concept of bisweep data to piecewise smooth domains, which yields a new non-Cauchy type partial data result for Caldern inverse conductivity problem. It is also shown that bisweep data are (up to a constant scaling factor) the Schwartz kernel of the relative Neumann-to-Dirichlet map. The results are based on the behaviour of the solutions of the conductivity equation under conformal mappings and the solvability of the Neumann problem for Laplace equation with measure boundary values.

Aku Seppänen (University of Eastern Finland)

Bayesian approach to tree detection based on airborne laser scanning data

Abstract: In this talk, the remote sensing of forest using airborne laser scanning (ALS) is considered. We propose a novel computational method for detecting trees on the basis of ALS data. In the proposed approach, locations, heights, and crown shapes of trees are tracked automatically by fitting multiple 3D crown height models to ALS data of a field plot. This estimation problem is written in the Bayesian inversion framework. The proposed formulation allows for utilizing prior information on tree shapes in the tree detection. Here, the prior models are written based on field measurement data and allometric models for tree shapes. The feasibility of the proposed approach is tested with ALS and field data from a managed boreal forest. With the new approach, 71.9 % of the trees in the area were detected, which is a clear improvement compared to a usual 2.5D crown delineation approach (53.1 % of the trees detected).

Samuli Siltanen (University of Helsinki)

Sparse Tomography

Abstract: A wavelet-based sparsity-promoting reconstruction method is studied in the context of tomography with severely limited projection data. Such imaging problems are ill-posed inverse problems, or very sensitive to measurement and modeling errors. The reconstruction method is based on minimizing a sum of a data discrepancy term based on an L^2-norm and another term comprising an L^1-norm of a wavelet coefficient vector. Depending on viewpoint, the method can be considered (i) as finding the Bayesian MAP estimate under white noise assumption and using a Besov space prior, or (ii) as deterministic regularization with a Besov norm penalty. The minimization is performed using a tailored primal-dual path following interior-point method, which is applicable to larger-scale problems than commercially available general-purpose optimization package algorithms. The method is parameter-free due to a novel technique called the "S-curve method," which can be used to incorporate a priori information on the sparsity of the unknown target to the reconstruction process. Numerical results are presented, focusing on uniformly sampled sparse-angle data. Both simulated and measured data are considered, and noise-robust and edge-preserving multiresolution reconstructions are achieved.

Stratos Staboulis (Aalto University)

Simultaneous Reconstruction of Outer Boundary Shape and Conductivity Distribution in Electrical Impedance Tomography

Abstract: The aim of electrical impedance tomography (EIT) is to reconstruct the admittance distribution inside a body from boundary measurements of current and voltage. In this work, the need for prior geometric information on the measurement setting of EIT is relaxed by introducing a Newton-type output least squares algorithm that reconstructs the admittance distribution and the object shape simultaneously. The functionality of the technique is demonstrated via numerical tests with experimental data.

Tanja Tarvainen (University of Eastern Finland)

Bayesian Image Reconstruction in Quantitative Photoacoustic Tomography

Abstract: This is joint work with Aki Pulkkinen, Ben T. Cox, Simon R. Arridge and Jari P. Kaipio. Photoacoustic tomography (PAT) is an emerging imaging modality developed over the last few decades which combines the benefits of optical contrast and ultrasound propagation. The optical methods provide information about the distribution of chromophores which are light absorbing molecules within the tissue such as haemoglobin, melanin and various contrast agents. The ultrasonic waves carry this optical information directly to the surface with minimal scattering, thus retaining accurate spatial information as well. Nowadays, PAT can be used to provide images of soft biological tissues with high spatial resolution. However, this information is only a qualitative image and it does not include information about the amount of chromophores. Quantitative photoacoustic tomography (QPAT) is a technique in which also the absolute concentration of chromophores is estimated. This is a hybrid imaging problem in which the solution of one inverse problem acts as a data for another ill-posed inverse problem. In this work, we consider the optical image reconstruction problem of QPAT in Bayesian framework.

Robert Winkler (Karlsruhe Institute of Technology)

Resistor network approaches to the Complete Electrode Model

Abstract: Determining the conductances of a resistor network from boundary measurements is a discrete analogon to Electrical Impedance Tomography. We show that the measurements of the Complete Electrode Model (CEM) match those of a resistor network in the homogeneous case and, following the approach of Borcea et.al., we determine the conductances and geometry of a resistor network that resembles the CEM both on the boundary and on a finite volume discretization of the interior.

Mykhaylo Yudyskiy (Johann Radon Institute for Computational and Applied Mathematics)

Fast wavelet-based methods in atmospheric tomography

Abstract: The problem of atmospheric tomography arises in ground-based telescope imaging with adaptive optics, where one aims to compensate in real-time for the rapidly changing optical distortions in the atmosphere. The mathematical formulation of the problem resembles limited angle tomography. The recent developments of wavelet-based reconstructions methods in atmospheric tomography are discussed. In this talk we give a short introduction to the topic, discuss the theoretical results, and present a few numerical examples.

Guo Zhang (University of Jyväskylä)

Reconstruction from boundary measurements for less regular conductivities

Abstract: In this paper, following Nachman's idea and Haberman and Tataru's idea, we reconstruct C^1 conductivity γ or Lipschitz conductivity γ with small enough value of $|\nabla \log \gamma|$ in the Lipschitz domain Ω from the Dirichlet-to-Neumann map Λ_γ.

Miren Zubeldia (University of Helsinki)

The Forward Problem for the Electromagnetic Helmholtz Equation

Abstract: In this talk we show recent results related to the uniform resolvent estimates and Sommerfeld radiation condition for solutions $u \in H^1_A(\mathbb{R}^d)$, $(d \geq 3)$ of the electromagnetic Helmholtz equation

$$(\nabla + iA(x))^2 u + V(x)u + \lambda u + i\varepsilon u = f$$

with singular potentials. We then deduce the limiting absorption principle, existence and uniqueness of the scattering cross-section and some spectral properties of the magnetic Schrödinger operator. We use a multiplier method and integration by parts as main tools.