This document has been created as part of the Erasmus+ project "Developing Digital Physics Laboratory Work for Distance Learning" (DigiPhysLab). More information: www.jyu.fi/digiphyslab

Coupled compass oscillations

Instructor version

25.2.2023

Co-funded by the Erasmus+ Programme of the European Union

Coupled compass oscillations - Instructor version

Task overview

- Topics: magnetism, magnetic moment, coupled oscillators, effective coupling factor
- Target group: Physics and physics teacher training students (>1st year)
- Time frame: > 4 hours to conduct experiments, analyze data, and write a report, but it is difficult to estimate the exact time of conducting the experiment and analyzing the data because it depends on the experimental skills of the student.
- The task can be conducted in a lab course, but students should be given time to complete the data analysis and write the report at home.
- It is recommended that students conduct the task independently at home, and then submit a report to the teacher (in written form or an oral presentation). If there are any difficulties, students can get instructor's help via e-mail or in a video meeting.
- If students have enough time, it is recommended to modify/expand the task with the additional analysis described at the end of this document.

In this task, students study the phenomena of coupled oscillations via the example of coupled oscillations of two magnetic dipoles, i.e., two magnetic needles. Students should determine the magnetic moment and moment of inertia of the needle, investigate how the effective coupling factor depends on the distance between the needles, and determine the beats frequency for one distance between the needles. The task isn't strictly guided and students don't know the "exact results" they need to obtain.

Required equipment

- Smartphone
- A computer with a video analysis software (e.g., *Tracker*) and data analysis (e.g., *Excel* and *SciDAVis*)
- Two equal magnetic needles on stands
- Paper protractor 360°

It is important that the friction between magnetic needles and the stand is as small as possible. The friction should be small enough that the magnetic needle can perform at least 5 to 6 full oscillations after it is removed from its equilibrium position.

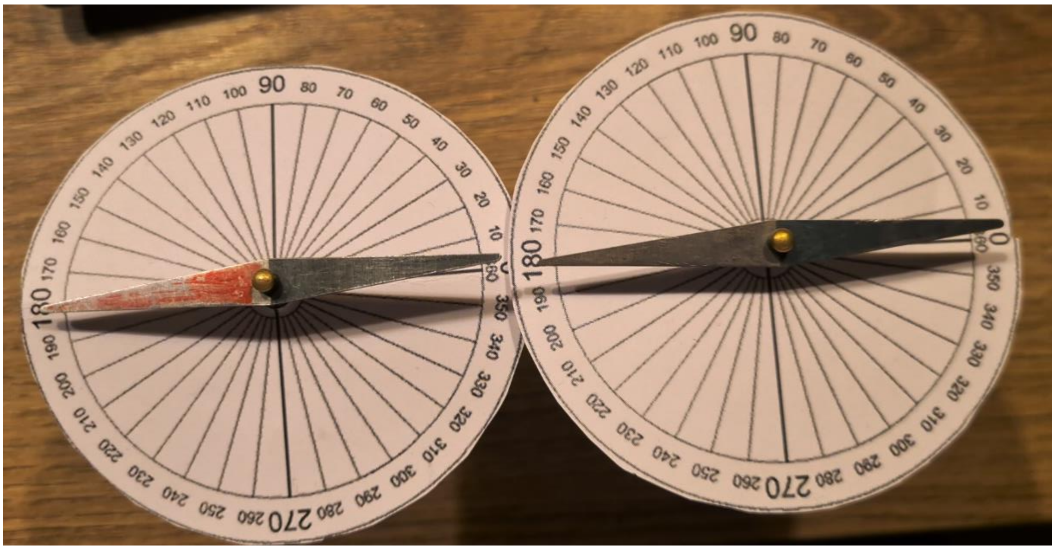


Figure 1. An example of an experimental setup. Two equal magnetic needles are located on equal stands. Paper protractors are placed on the stands so that students can determine from which position they will let the needles to oscillate.

During our task pilots, students have used *Tracker* for video analysis, and *Excel* and *SciDAVis* for data analysis. Instructions for using the three listed software can be found on our website in other task documents.

Pre-lab exercise

During the task, students will have to determine the frequency of different oscillations. They will record oscillations with a phone and then determine the period duration and frequency by using video analysis. If the students don't know how to use any video analysis software, it is recommended that they do a simple pre-lab exercise before they start conducting the task. The pre-lab exercise contains instructions for using *Tracker* that they will need to conduct the task. If the students already know how to use some video analysis software, they can use the software of their choice.

Comments and suggestions for conducting the task

The idea is that the students receive task instructions and the necessary equipment (two magnetic needles on the stands and 2 paper protractors) and independently conduct the experiment and data analysis at home.

1. Determination of the magnetic moment of the magnetic needle

In this part of the task, in order to determine the magnetic moment of the needle, students need to determine the fundamental frequency and the moment of inertia of the magnetic needle and find the value of the horizontal component of the Earth's magnetic field for the place/city where they are conducting the experiment.

- No magnetized object should be near the needle.
- It is always better to measure the time of several oscillations and calculate the period duration from that time than to measure one period.
- To calculate the moment of inertia, the needle is modeled as a thin rectangular plate. Given that the needles do not have everywhere the same width, it is necessary to determine which width will be taken for calculation. Students should take a width greater than the average width of the needle because most of the mass of the needle is positioned close to the axis of rotation.

2. Determination of the dependence of the effective coupling constant on the distance between the magnetic needles

In this part of the task students should achieve in phase oscillations of needles and oscillations in phase opposition for several different distances between the needles. From the frequencies of oscillations in phase and phase opposition, they can calculate the effective coupling factor for each distance. From the measured data they need to derive the dependence of the effective binding constant on the distance between the magnetic needles.

Data collection and analysis

Students need to position the needles so that their axes in the equilibrium position coincide. For oscillations in phase, they deflect both needles from the equilibrium position for the same angle in the same direction and for phase opposition oscillations they deflect the needles for the same angle in opposite directions. The deflection angle should not exceed 30°. From the introductory theory, students should conclude on their own what initial conditions for the oscillation they should establish in each experiment. Students use *Tracker* to determine the frequency of the needle oscillating in phase and in phase opposition at a certain distance between the needles. Then, using these frequencies and the moment of inertia of the needles, they calculate the effective coupling factor for that distance. They need to repeat measurements for at least 6 distances.

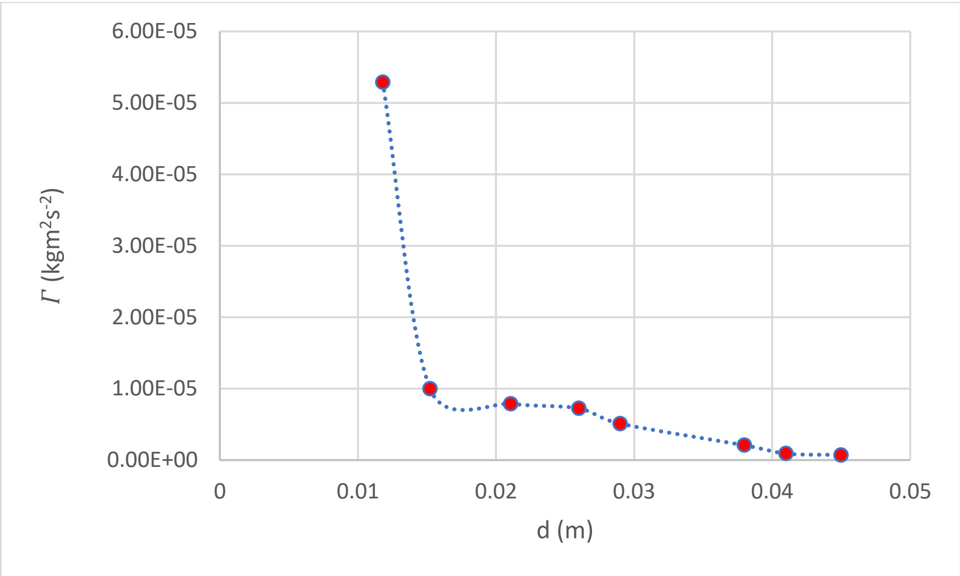


Figure 2. An example graph of the dependence of the effective binding constant to the distance between magnetic needles.

The dependence is assumed to be described with

$$\Gamma = A \cdot d^x,$$

where Γ is the effective coupling factor, d the distance between the magnets, and A and x are the free parameters. Logarithmization yields a linear equation:

$$\log(\Gamma) = x \cdot \log(d) + \log(A).$$

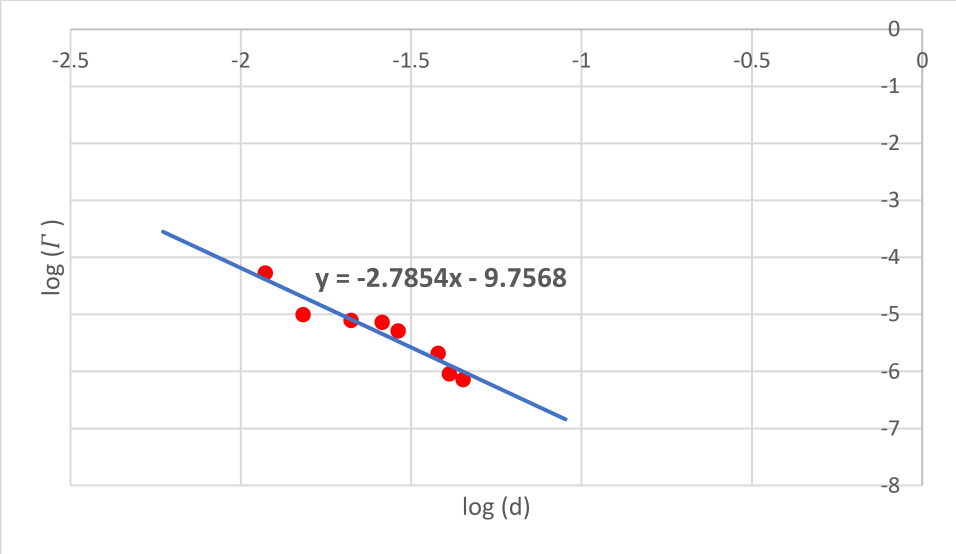


Figure 3. An example of a graph of the dependence of the effective binding constant to the distance between magnets shown on the logarithmic scale.

The effective coupling factor decreases with the third power of the distance between the magnets. Figure 3 shows an example of an experimental result that is approximate to the theoretical value. The frequency of phase opposition oscillations is greater than the frequency of in phase oscillations for each of the distances. By increasing the distance between magnetic needles i.e., reducing the effective coupling factor, the frequencies of in phase oscillations and oscillations in phase opposition are equalizing, i.e., approaching the fundamental frequency.

3. Betas frequency

In the last part of the task, students need to analyze the beats of the magnetic needles. During the beats, the amplitude of the needle gradually decreases from its maximum value until it reaches zero, then the amplitude gradually increases to the maximum value, then decreases until it reaches zero, and then it increases again to the maximum value which takes the needle back to the starting point of its motion. This behavior is repeated until the frictional force stops it. Students estimate the beats frequency by the video analysis. The beats period is the time that passes during the whole cycle described above. In the task pilot student estimated the beats period as the time that passes between two consecutive needle stops, which is actually time $T/2$. That led to a discrepancy between the measured and theoretical value. The theoretical value of the beat's frequency is calculated from the expression

$$\omega_b = \frac{\sqrt{\omega_0^2 + 3\Omega^2} - \sqrt{\omega_0^2 + \Omega^2}}{2}.$$

This equation should be added to the students' version of the task along with the short description of the beats if students have no prior knowledge about beats. Students need to compare the measured with the theoretical value and discuss reasons for possible differences.

Possible modification/extension of the task

The last part of the exercise can be modified so that the students calculate the beats frequency instead of just estimating it. They obtain the beats frequency from the change of the deflection angle in time. Students need to draw a graph of the angle over time for one needle performing the beats. Then they need to fit the graph with the function

$$\theta(t) = Ae^{-\gamma t}\sin(\omega t + \varphi_1)\sin(\omega_b t + \varphi_2),$$

where A is the amplitude, γ the damping coefficient, ω the frequency of oscillations, ω_b the beats frequency, and $\varphi_1$and φ_2 the phase angles of the two needles. This function takes into account the decrease in the amplitude that occurs due to friction. From this fit students can derive other unknown parameters like the damping coefficient in addition to the beats frequency.

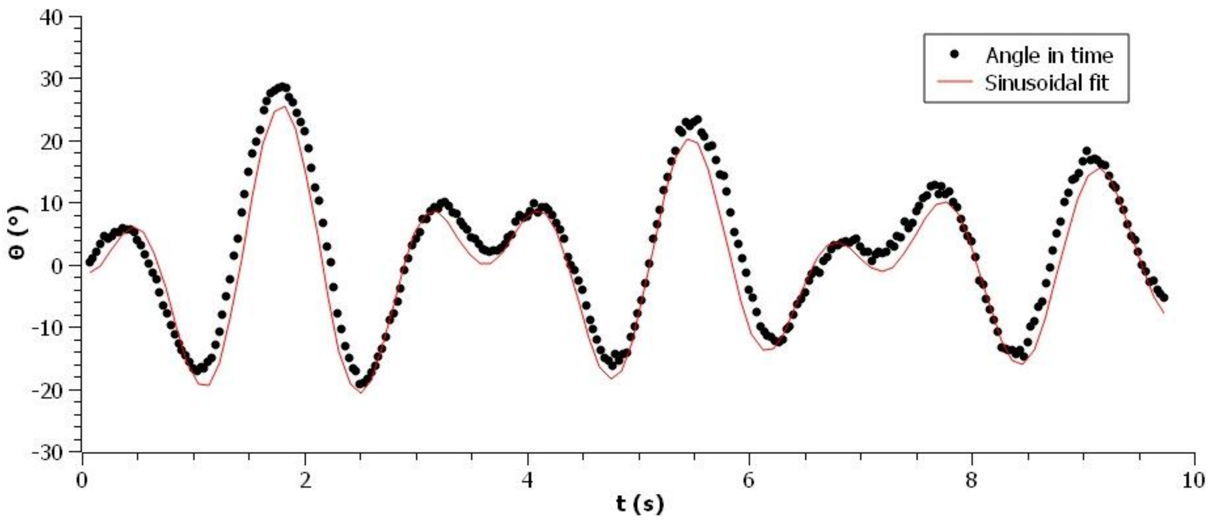
Data about the dependence of the angle of the needle on time can be obtained in *Tracker*. Students can track the motion of the magnetic needle over time using the option *Autotracking*. Extended instructions for using *Tracker*, in which the tracking procedure is described step by step, can be found in the attachment of the exercise "Sliding smartphone" on our website.

In the app, students should select the part of the one needle that they want to track and then the app can find and mark the position of the selected part of the object at any time. After completing the tracking, data on the deflection angle in time can be found in *Tracker*. The data can be copied and analyzed in any data analysis software.

While recording the beats, students should take special care that the phone is not moving, because the movement can affect the angle of deflection that is detected in the video.

When analyzing the video, the coordinate system should be positioned so that the origin is in the center of the magnetic needle whose motion they choose to track, and the x-axis is parallel to the needle when it is in the equilibrium position.

If students don't know how to use any software in which they can fit the measured data to the given sine function, they can be given instructions for using the *SciDAVis* application that can be found in the attachment to the "Elevator oscillations" task on our website.



```
[Wednesday, February 15, 2023 5:42:01 PM Central European Standard Time        Plot: "Graph3"]
Non-linear fit of dataset: Table1_angle, using function: A*exp(-g*x)*sin(w*x+f1)*sin(o*x+f2)
Y standard errors: Unknown
Scaled Levenberg-Marquardt algorithm with tolerance = 0.0001
From x = 0.067 to x = 9.732
A  = 28.7982213646952 +/- 1.0064031687438
g  = 0.0616486783723858 +/- 0.00686953592002057
w  = 4.26720676295933 +/- 0.00657126060997928
f1  = 0.199735166652301 +/- 0.0335127844309465
o  = 0.912117644450516 +/- 0.00686642491197265
f2  = -0.157869697010226 +/- 0.0304451948711375
---------------------------------------------------------------------------------
Chi^2 = 3,242.01509653439
R^2 = 0.914782866953481
```

Figure 4. An example of fitting the graph of angle over time to a given sine function from the task pilot. All unknown parameters are listed below the graph, where A is the amplitude, g the damping coefficient, w frequency of oscillations, o the beats frequency, and f1 and f2 the phase angles of the two needles.