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Coupled compass oscillations

Student version

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Motivation

Oscillations are one of the most common motions that can be found in nature. One can come across many examples where two or more oscillating systems are coupled together e.g. oscillations of bound atoms within a molecule . In physics lessons, usually the example of two pendulums connected with an elastic spring is studied. Although we will not encounter objects connected with an elastic spring in nature, studying this example is useful as a large number of coupled oscillations can be modeled as oscillations of objects connected with an elastic spring. One such example is the coupled oscillation of magnetic dipoles. Here, the magnetic force takes over the role of the elastic spring as the interaction between the two dipoles. Dipole-dipole interactions are a common phenomenon at the microscopic level. An example from magnetism is the interaction between nuclei and electrons in atoms and molecules, whose dipole moments arise from their intrinsic spins. The aim of this task is to investigate the coupled oscillations of two magnetic dipoles.

A magnetic needle is an example of a magnetic dipole at the macroscopic level. Each dipole near the Earth feels the Earth's magnetic field, which is approximately homogeneous on the surface of the planet. If we place two needles at a sufficiently small distance, each of them will be influenced by the field of the other needle along with the Earth's magnetic field. A dipole-dipole interaction will be established and if we cause the needles to oscillate, their oscillations will be coupled.

The formalism is analogous to the one for spring-connected pendulums. **We assume that the needle axes in the equilibrium position coincide and that their displacement angles are small.** We analyze three cases for three different initial conditions:

1) In-phase oscillations:

At $t = 0$, needles are displaced from the equilibrium position by the same angle $\theta_1 = \theta_2 = \theta_A$. Then the deflection angle in time is described by:

$$\theta_1(t) = \theta_2(t) = \theta_A \cos\left(\sqrt{\omega_0^2 + \Omega^2} \cdot t\right) \tag{1}$$

2) For oscillations in phase opposition:

At $t = 0$, one needle is displaced from the equilibrium position by the angle $\theta_1 = -\theta_A$, and the other one for the angle $\theta_2 = \theta_A$. Then the deflection angles in time are described by:

$$\theta_1(t) = \theta_A \cos\left(\sqrt{\omega_0^2 + 3\Omega^2} \cdot t\right) \tag{2}$$

$$\theta_2(t) = -\theta_A \cos\left(\sqrt{\omega_0^2 + 3\Omega^2} \cdot t\right) \tag{3}$$

3) Beats

At $t = 0$, one needle is in the equilibrium position $\theta_1 = 0$, and the other is deflected by the angle $\theta_2 = \theta_A$. Then the deflection angles in time are described by:

$$\theta_1(t) = \theta_A \cos\left(\frac{\sqrt{\omega_0^2 + 3\Omega^2} - \sqrt{\omega_0^2 + \Omega^2}}{2} \cdot t\right) \cdot \cos\left(\frac{\sqrt{\omega_0^2 + 3\Omega^2} + \sqrt{\omega_0^2 + \Omega^2}}{2} \cdot t\right) \quad (4)$$

$$\theta_2(t) = -\theta_A \sin\left(\frac{\sqrt{\omega_0^2 + 3\Omega^2} - \sqrt{\omega_0^2 + \Omega^2}}{2} \cdot t\right) \cdot \sin\left(\frac{\sqrt{\omega_0^2 + 3\Omega^2} + \sqrt{\omega_0^2 + \Omega^2}}{2} \cdot t\right) \quad (5)$$

In all three cases, we assume that the friction between the needles and the surface is negligible. In these equations, ω_0 is the fundamental frequency of the magnetic needle oscillating in the Earth's magnetic field described by the expression

$$\omega_0^2 = \frac{\mu B}{I}$$

where μ is the magnetic moment of the needle, B the horizontal component of the Earth's magnetic field, and I the moment of inertia of the needle. Ω^2 however, is an abbreviation for

$$\Omega^2 = \frac{\Gamma}{I}$$

where Γ is the effective coupling factor of the needles analogous to the elastic force $F_{el} = -kx$ for spring-coupled objects.

The effective coupling factor can be calculated from frequencies of in-phase oscillations and oscillations in phase opposition. From expression (1) follows that the frequency of an in-phase oscillation is $\omega_1^2 = \omega_0^2 + \Omega^2$, and from expression (2) follows that the frequency of an oscillation in phase opposition is $\omega_2^2 = \omega_0^2 + 3\Omega^2$. Combining these expressions leads to

$$\Gamma = \frac{I(\omega_2^2 - \omega_1^2)}{2}$$

Pre-lab exercise

Read the instructions for using *Tracker* and solve the set of tasks there.

Equipment list

Smartphone, computer with *Tracker* and data analysis software, 2 equal magnetic needles on stands, paper protractor 360 °

Experimental skills in focus

Planning an experiment, data collection, and analysis

Task description

1. Place one magnetic needle in a location where the Earth's magnetic field is the only magnetic field that significantly affects the needle. Deflect the needle from the equilibrium position and record its oscillation with the camera of your phone.

   a. Determine the period of the oscillation via video analysis in *Tracker*. Calculate the fundamental frequency ω_0 of the magnetic needle.

b. Calculate the moment of inertia of the magnetic needle. Model the needle as a thin rectangular plate. You can measure the plate dimensions in *Tracker*. *Note:* Estimate what width of the needle is the best to be taken for calculation so that your model is as close as possible to the actual object.

c. On the official NOAA website (https://www.ngdc.noaa.gov/geomag/magfield.shtml), you can find the value of the horizontal component of the Earth's magnetic field for the place/city where you are conducting the experiment.

d. Calculate the magnetic moment of the needle.

2. In this task, you will investigate how the effective coupling factor depends on the distance between the magnetic needles.

   a. Place the second needle near to the first one and let them oscillate in phase. Record the oscillations again with your smartphone and determine the frequency in *Tracker*. Keep the needles at the same distance, and by using *Tracker*, measure how far the needles are from each other when they are in the equilibrium position.

   b. Let the needles oscillate in phase opposition at the same distance as before. Record the oscillations and determine the frequency in *Tracker*.

   c. Calculate the effective coupling factor.

   d. Repeat the measurements for at least 5 more distances between the needles and determine the effective coupling factor for each distance. For one of these distances, let the needles oscillate so that beats occur. Record that oscillation and save the video for later analysis.

   e. Determine how the effective coupling factor depends on the distance between the magnetic dipoles.

3. Use the video of the beats you already recorded before to determine (estimate) the beat frequency. Using equation (4) determine the theoretical value of the beat frequency for the distance at which you recorded the beats. Compare the measured and theoretical value of the beat frequency.

Assessment

Write a report according to the task instructions. For each part of the task, describe briefly how you conducted the experiment and how you analyzed the data. Discuss the limitations of the experiments. Draw conclusions based on the results of the experiments and discuss them. Comment what happens in the limits of weak and strong coupling. Base your arguments on your data and its graphical representations.