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Digital Signal Processing

Student version

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Motivation

Imagine you have just tasted the most amazing food in the world. You wish to know the recipe for it, but there is no-one who can tell you what the ingredients are. You can perhaps figure out a few by eye, and some more by tasting, but it is hard to know for certain. Now imagine that, even in the case of the most smoothly blended awesomesauce, you could sample a spoonful, and after some milliseconds you could tell not only which ingredients went in, but also their proportions. For a digital signal, to an extent, we have just that in what is called the discrete Fourier transform (DFT).

Apologies for making you hungry with the food analogy, but it gives an idea of how powerful Joseph Fourier's work has been for digital signal processing. The idea behind the Fourier transform is that mostly any function can be represented as a sum of sinusoids, so precisely that only mathematicians need to care about the difference.

Let's define a signal $\{x_n\} = \{x_0,\ x_1,\ \dots,\ x_{N-1}\}$ consisting of N data points (samples) taken at constant time intervals of T_s seconds each. The DFT of the signal is computed as

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}, \tag{1}$$

where each X_k is a complex number, and the set of $\{X_k\} = \{X_0, X_1, \dots, X_{N-1}\}$ represents the signal in the frequency domain. For a continuous function this would correspond to a transformation of a function of time to a function of frequency. The interpretation of the Fourier-transform coefficients X_k can become clearer when we look at the inverse DFT:

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{\frac{i2\pi}{N}kn}, \tag{2}$$

where we can see that the original signal can be represented as a sum of complex sinusoid components, and X_k describes the amplitude and phase of each component. In this experimental task we utilize the amplitude spectrum of the Fourier transform defined for a real-valued signal as

$$A_j = \frac{2}{N}|X_j|, \qquad j = \ 0, 1, \dots, N/2 \tag{3}$$

which tells us how strongly the frequency f_j is present in the signal, with $\{f\}_j = \{f_0, f_1, \dots, f_{N/2}\}$, where the frequencies are related to the sampling frequency $f_s = \frac{1}{T_\text{s}}$ as

$$f_j = \frac{jf_s}{N} = \frac{j}{NT_\text{s}}, \tag{3}$$

so that the frequencies visible in the Fourier transform never exceed $f_s/2$. Note that a real-valued signal of N samples in the time domain corresponds to $N/2 + 1$ physically meaningful frequencies in the frequency domain.

In this experimental task, we will not focus on the mathematical subtleties of the discrete Fourier transform, so do not worry if the above definitions are not instantly clear to you. The interactive instructions in a python notebook will guide you via several examples. We will aim for an understanding of what the transform can tell us, and how to apply the transform on data collected on a smartphone (or on any other discrete signal). More details can be found in the supplementary material given at the end of this document.

Equipment list

Smartphone with the app phyphox (RWTH Aachen University) or some other app giving access to accelerometer data, a computer for running and editing the online python notebook/scripts provided with the instructions.

Experimental skills in focus

Digital signal processing, data analysis, data collection

Task description

In this task we get to know the basics of what the discrete Fourier transform can do and how it is applied to a signal. We will apply the DFT to our collected data to answer a question we set out to inquire experimentally. Follow the instructions on the accompanying python notebook (also available at https://jyu.fi/digiphyslab), and ultimately perform an experiment to measure one of the following:

1. The vibration frequency of your phone
2. The spin-dry rotation frequency of your washing machine (but do not put your phone inside the machine)
3. The vibration frequency of a computer (if strong enough to be measured).
4. The frequency of vibration inside a car due to the running engine
5. Your heart rate
6. The frequency of some other periodic signal that you come up with yourself

Assessment

Write a brief computational essay, consisting of text, figures, and snippets of code and code output, in which you

a) answer the **bolded** questions in the python notebook instructions and show the requested figures.
b) describe what you set out to investigate and explain the used experimental setup.
c) show the code that you used for the data analysis of your experiment.
d) discuss the obtained results and the uncertainties involved in the measurements.
e) reflect on the experiment. Write down ideas that for you were the most meaningful during this experimental task. Think also of possible ideas for how to revise your experiment.

The computational essay should tell a coherent story. The task instruction notebook is an example of what a computational essay looks like. You can create a copy of the task instruction notebook and build your essay in the same notebook format.

Supplementary material

- Steven W. Smith, “The Scientist and Engineer's Guide to Digital Signal Processing”, Second Edition, California Technical Publishing, 1999. Available online for free: https://www.dspguide.com/
- D. Donnelle and B. Rust, "The fast Fourier transform for experimentalists. Part I. Concepts" in Computing in Science & Engineering, vol. 7, no. 2, pp. 80-88, 2005. Available online for free: https://ieeexplore.ieee.org/document/1401806