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How to model a bottle?

Student version

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# How to model a bottle?

## Motivation

Many musical instruments, such as the flute, clarinet, or trumpet, rely on a vibrating column of air inside the instrument to produce sound. The pitch (dominating sound frequency) of the instrument is then changed by altering the length of the vibrating air column. In this experiment we will examine a simple instrument: a bottle with various amounts of water in it. By blowing across the top of the bottle one can create a steady pitch from the bottle, and by adding water one can change the pitch. Many flutes, for example, produce their sound by a similar blowing technique, although their way of changing the pitch is somewhat more subtle.

The bottle is often modeled as a tube that is closed at one end (the bottom) and open at the other (the top), see Figure 1. The lowest frequency of standing waves permitted by such open-closed tube is of the familiar form

|  |  |  |
| --- | --- | --- |
|   | $$f = \frac{c\_{air}}{4L} ,$$ | (1)  |

where $c\_{air}$ is the speed of sound in air, and $L$ is the length of the air column in the tube.



Figure 1: The lowest frequency permitted by an open-closed tube. The wavelength of the forming standing wave is four times the length of the tube.

A Helmholtz resonator is a bottle-like object where a closed spherical cavity of volume $V$ is connected to open air via a cylindrical neck of length $L\_{neck}$, see Figure 2. The principle of a Helmholtz resonator is that the air in the neck can oscillate against the springiness of the air in the cavity, creating an oscillation with a resonance frequency (see the bottom of the document for the idea of the derivation)

|  |  |  |
| --- | --- | --- |
|   | $$f\_{H}=\frac{c\_{air}}{2π}\sqrt{\frac{A}{VL\_{neck}}} ,$$ | (2)  |

where $A$ is the cross-sectional area of the opening of the neck. Compare equations (1) and (2). What do they have in common and what is different?



Figure 2: A Helmholtz resonator.

In this task we will design an experiment to put the models of Eqs. (1) and (2) to the test.

**The main learning objective of this experiment is to practice comparing model predictions to data.**

## Equipment list

Smartphone (frequency analyzer), at least one bottle (long neck preferred), water, a measuring cup of one dl (100 ml) or similar, ruler or tape measure, graphing and analysis software. Spreadsheet software will also be useful. You can access the frequency data by using the free smartphone app phyphox (RWTH Aachen University) or Physics Toolbox Sensor Suite (Vieyra Software / Chystian Vieyra).

## Experimental skills in focus

Planning an experiment, data collection, comparing model predictions with data, data representation.

## Safety

It might take a few tries to get a solid sound from the bottle. Take breaks from blowing every now and then to avoid overbreathing.

## Task description

Test the models of an open-closed tube and a Helmholtz resonator for the frequency of the sound of blowing across a bottle top, given in Equations (1) and (2), respectively. Pay attention to the domain in which each model is applicable and attempt also to probe the limits of the models. Plan and execute the required measurements. Choose a suitable way to represent your collected data and make a comparison between your data and the predicted frequencies given by the two models. Remember to estimate the experimental uncertainty of your results.

**Extra investigation:**

One can refine the models behind Equations (1) and (2) by introducing the so-called effective length. Due to the finite radius of the tube or bottle neck, the vibrating length of air extends beyond the length of the cylindrical tube. This can be accounted for by adding a correction proportional to the radius of the bottle neck to the lengths in Equations (1) and (2):

|  |  |  |
| --- | --- | --- |
|   | $$L\_{(neck)}^{eff}=L\_{(neck)}+ ar ,$$ | (3)  |

where $a$ is a dimensionless parameter and $r$ is the radius of the tube (here bottle neck). Note that $ a$ can have different values for each model. You can also look for suitable values from literature.

If there is time, explore the possibility of refining the models by using an effective length as in Equation (3). Given even more time and equipment, how could you further test the two models?

## Assessment

Create a short presentation (roughly 5 PowerPoint slides, for example) describing your work and findings. Focus on delivering a conclusion about the validity of each model backed up by your data in a suitable representation.

**Derivation of the Helmholtz resonance frequency (optional exercise):**

Let there be mass *m* of air in the bottle neck. If the air in the neck is displaced by a small length δ*γ*, the volume of air in the cavity becomes $V + δV$ with a pressure $p + δp$. Assume that this is an adiabatic process and show that

|  |  |  |
| --- | --- | --- |
|   | $$\frac{δp}{p}=-γ\frac{δV}{V},$$ |  |

where *γ* is the adiabatic index. Next, write down Newton’s II law for the mass of air in the bottle neck. You should find a differential equation of a harmonic oscillator. Use the above result to derive Equation (2) for the resonance frequency of a Helmholtz resonator.

Hints: Remember that $c=\sqrt{γ\frac{p}{ρ}}$ for the speed of sound in a medium with density $ρ$.