INVERSE PROBLEMS SESSION (FRI 9/1/26)

ROOM C5 (12.45-15.35)

Giovanni Covi (University of Jyväskylä)
Inverse problems for the viscoacoustic wave equation **(12.45-13.05)**

We study the viscoacoustic wave equation, which has a time-convolutional term in addition to the usual wave operator. This equation is used for modeling an elastic medium with memory, in which the stress depends on all the history of the gradient of the deformation. We will show that it is possible to recover both the wave speed and the kernel of the memory term under convenient geometric assumptions. We will do so via a Gaussian beam quasimode construction and a propagation of singularities argument. As an application, we introduce the extended Maxwell model, which represents the viscoelastic behaviour of a polymeric medium by means of a microscopic parallel array of spring/dashpots elements. We show how our theoretical result can be applied to the recovery of the physical parameters of the medium. This is an ongoing joing project with Maarten de Hoop and Mikko Salo.

Spyros Filippas (University of Helsinki)
On the stability of a hyperbolic inverse problem **(13.15-13.35)**

The Boundary Control method is one of the main techniques in the theory of inverse problems. It allows to recover the metric or the potential of a wave equation in a Riemannian manifold from its Dirichlet to Neumann map (or variants) under very general geometric assumptions. In this talk we will address the issue of obtaining stability estimates for the recovery of a potential in some specific situations. As it turns out, this problem is related to the study of the blow-up of quantities coming from control theory and unique continuation. This is based on joints works with Lauri Oksanen.

Seán Gomes (University of Helsinki)
Lorentzian Calderón problem on vector bundles **(13.45-14.05)**

We discuss a version of the Calderón problem for the connection Laplacian

$$P = \nabla^*\nabla + V(t,x)$$

acting on sections of a Hermitian vector bundle E over a fixed Lorentzian manifold (M,g). Under suitable geometric assumptions on (M,g), we show that the connection ∇ and potential V are uniquely determined by the Dirchlet–to–Neumann map up to the natural group of gauge transformations. In particular, the result is applicable to (M,g) that are small perturbations of Minkowski geometry. This investigation builds on earlier works in the scalar setting by Alexakis–Feizmohammadi–Oksanen.

Marvin Knöller (University of Helsinki)
A Computational Method for the Inverse Robin Problem with Convergence Rate **(14.15-14.35)**

In this talk we consider the inverse Robin problem, which is the determination of the Robin parameter a appearing in an elliptic partial differential equation's boundary condition. Let $\Omega \subset \mathbb{R}^2$ be bounded and sufficiently regular. Suppose that we know the solution to the Robin problem

$$\begin{aligned} \Delta u &= f \quad \text{in } \Omega\,, \\ \partial_\nu u + au &= g \quad \text{on } \partial\Omega \end{aligned}$$

only on a small subdomain $\omega \subset \Omega$ as well as the right hand sides $f \in L^2(\Omega)$ and $g \in H^{1/2}(\partial\Omega)$. Under the main assumption that the Robin parameter lies in a finite dimensional space of continuously differentiable functions, we establish a Newton-type algorithm for its reconstruction. This reconstruction algorithm requires first order piecewise continuous finite elements only. We show that the algorithm converges quadratically in the finite element's mesh size to the unknown Robin parameter. Moreover, we establish a perturbation analysis. We study several numerical examples that highlight the efficacy of our approach. Furthermore, we verify our theoretical convergence rates numerically.

This talk is based on joint work with Erik Burman (University College London) and Lauri Oksanen (University of Helsinki).

Antti Kykkänen (Rice University)
Horizontal and vertical regularity of elastic wave geometry **(14.45-15.05)**

The elastic properties of a material are encoded in a stiffness tensor field and the propagation of elastic waves is modeled by the elastic wave equation. In this talk characterize analytic and algebraic properties a general anisotropic stiffness tensor field has to satisfy in order for Finsler-geometric methods to be applicable in studying inverse problems related to imaging with elastic waves. The talk is based on joint work with Joonas Ilmavirta and Pieti Kirkkopelto.

Lisa Schätzle (Aalto University)
An explicit reconstruction formula for inverse Born scattering **(15.15-15.35)**

We consider the inverse medium scattering problem for the Helmholtz equation in two dimensions, i.e., the task to recover a compactly supported penetrable two-dimensional scatterer, modeled by a contrast function, from full knowledge of the associated far field data or, equivalently, of the far field operator. Although this problem is uniquely solvable, it is (1) severely ill-posed as small perturbations in the observed far field data may lead to large reconstruction errors and (2) nonlinear. In the regime of weak scattering, the Born approximation yields a linearized relation between the contrast and the far field data and thus overcomes the second difficulty of nonlinearity. This linear setting allows us to build on recent work for linearized EIT, which relies on a triangular Zernike decomposition, to derive an explicit reconstruction formula that expresses the expansion coefficients of the contrast in terms of those of the far field data. By choosing the expansion functions appropriately, the resulting system matrix decouples in angular direction and becomes lower triangular for each angular frequency separately. Consequently, each of these systems can independently be solved by performing a forward substitution. In this talk, we derive the resulting reconstruction formula and show numerical examples. Remarkably, our numerical experiments indicate that this formula together with an adequate regularization method remains effective even when applied to full nonlinear far field data beyond the Born regime. This is a joint work with Nuutti Hyvönen.