Cerium Halo Structures within Relativistic Hartree-Fock-Bogoliubov theory

Long WenHui (龙文辉)
School of Physics, Peking University, China
Physik-Department der Technischen Universität München, Germany

OUTLINE

I. Introduction and Motivation
II. Density dependent RHFB theory \textit{arXiv: 0812.1103v2 [nucl-th]}
III. Results and discussions: Halos in Cerium isotopes
IV. Conclusion and Perspective

Cooperators

Prof. Peter Ring: Physik-Department der Technischen Universität München, Germany
Prof. Nguyen Van Giai: Institut de Physique Nucleaire, Universite Paris-Sud XI, France
Prof. Meng Jie (孟杰): School of Physics, Peking University, China
Introduction

Nuclear Halo: exotic mode in EXOTIC NUCLEI

1. Special feature: extremely diffuse nuclear matter distribution \(^{11}\text{Li}\) [I. Tanihata(1985)]

2. Strong enhancement on reaction cross section [I. Tanihata(1985)]

3. Reduction of shell structure with halo occurrence: 
   \(N = 8\) (Li, Be), \(N = 50\) (Ca), \(N = 82\) (Zr)

4. Giant halos \((N_{\text{halo}} > 2)\): \(^8\text{He}, ^{14}\text{Be}; ^{14}\text{Zr}\) [J. Meng(1998)]

5. Candidate for BCS-BEC crossover [K. Hagino(2007)]

6. Representatives: \(^{11}\text{Be}, ^{19}\text{C}; ^6\text{He}, ^{11}\text{Li}, ^{17}\text{B}, ^{19}\text{B}, ^{22}\text{C}; ^8\text{He}, ^{14}\text{Be}; ^{8}\text{B}, ^{26}\text{P}; ^{17}\text{Ne}, ^{27}\text{S}\)
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2. Halo Formation: low-\( l \) states nearby the particle continuum threshold
3. Stability mechanism: pairing correlations (continuum effects, shell structure, level density, ...)
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- **Significant improvements brought by the exchange terms**
  1. Improved isospin dependence on effective mass \(\text{[W.H. Long(2006)]}\)
  3. *Tensor \(\rho\): eliminate unphysically large gaps at \(N, Z = 58\) and 92* \(\text{[W.H. Long(2007)]}\)
  4. *Better description of Neutron star properties: e.g., critical mass \(\sim 1.45M_\odot \text{ (1.5}M_\odot\text{)}* \(\text{[B.Y. Sun (2008)]}\)
  5. *Fully self-consistent* charge-exchange relativistic RPA based on DDRHF \(\text{[H.Z. Liang(2008)]}\)
RHF Hamiltonian  A. Bouyssy (1987)

- Hamiltonian of the system:

\[
H = \sum_{\alpha \beta} c_\alpha^\dagger c_\beta T_{\alpha\beta} + \frac{1}{2} \sum_{\alpha\alpha'\beta\beta'} c_\alpha^\dagger c_\beta^\dagger c_{\beta'} c_{\alpha'} \sum_\phi V_{\alpha\beta\alpha'\beta'},
\]

Kinetic term: \(T_{\alpha\beta} = \int d\mathbf{x} \bar{\psi}_\alpha (-i\gamma \cdot \nabla + M) \psi_\beta\)

two-body terms: \(V_{\alpha\beta\alpha'\beta'}^\phi = \int d\mathbf{x} d\mathbf{x}' \bar{\psi}_\alpha(\mathbf{x}) \bar{\psi}_\beta(\mathbf{x}') \Gamma_{\phi}(\mathbf{x}, \mathbf{x}') D_{\phi}(\mathbf{x}, \mathbf{x}') \psi_{\beta'}(\mathbf{x}') \psi_{\alpha'}(\mathbf{x}).\)

- \(\Gamma_{\phi}(\mathbf{x}, \mathbf{x}')\): Two-body interaction matrices

\[
\Gamma_\sigma \equiv -g_\sigma(x)g_\sigma(x') \quad \Gamma_A \equiv \frac{e^2}{4} (\gamma_\mu(1 - \tau_3))_x (\gamma^\mu(1 - \tau_3))_{x'}
\]
\[
\Gamma_\omega \equiv (g_\omega \gamma_\mu)_x (g_\omega \gamma^\mu)_{x'} \quad \Gamma_\pi \equiv \frac{-1}{m_\pi^2} (f_{\pi} \overline{\tau} \gamma_5 \gamma_\mu \partial^\mu)_x \cdot (f_{\pi} \overline{\tau} \gamma_5 \gamma_\nu \partial^\nu)_{x'}
\]
\[
\Gamma_{\rho}^V \equiv (g_\rho \gamma_\mu \overline{\tau})_x \cdot (g_\rho \gamma^\mu \overline{\tau})_{x'} \quad \Gamma_{\rho}^T \equiv \frac{1}{4M^2} (f_\rho \sigma_{\nu k} \overline{\tau} \partial^k)_x \cdot (f_\rho \sigma_{\nu l} \overline{\tau} \partial^l)_{x'}
\]
\[
\Gamma_{\rho}^{VT} \equiv \frac{1}{2M} (f_\rho \sigma^{k\nu} \overline{\tau} \partial_k)_x \cdot (g_\rho \gamma_\nu \overline{\tau})_{x'} + (g_\rho \gamma_\nu \overline{\tau})_x \cdot \frac{1}{2M} (f_\rho \sigma^{k\nu} \overline{\tau} \partial_k)_{x'}
\]

- \(D_{\phi}(\mathbf{x}, \mathbf{x}')\): Propagators for meson (\(\sigma, \omega, \rho, \pi\)) and photon (\(A\)) fields

- Energy functional: expectation of Hamiltonian with the no sea approximation
In-medium effects: DD Meson-Nucleon couplings

- **Density dependent (DD) meson-nucleon couplings** [W.H. Long(2005~2007)]


\[
g_i(\rho_b) = g_i(0)e^{-a_i\xi}, \quad g_i = g_\rho, f_\rho, f_\pi, \quad a_i = a_\rho, a_T, a_\pi; \quad (8)
\]

\[
g_i(\rho_b) = g_i(\rho_0)f_i(\xi), \quad i = \sigma, \omega \quad (9)
\]

where \( \xi = \rho_b/\rho_0 \), and

\[
f_i(\xi) = a_i \frac{1 + b_i(\xi + d_i)^2}{1 + c_i(\xi + d_i)^2}; \quad \text{with } f_i(1) = 1, f_i''(0) = 0, f_\sigma''(1) = f_\omega''(1) \quad (10)
\]

2. Rearrangement terms \( \Sigma^\mu_R \) induced by density dependence

\[
\Sigma \rightarrow \Sigma + \gamma_\mu \Sigma^\mu_R, \quad (11)
\]

where \( \Sigma_R = \Sigma_{R,\sigma} + \Sigma_{R,\omega} + \Sigma_{R,\rho} + \Sigma_{R,\pi} \).
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      where \( \xi = \rho_b / \rho_0 \), and

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      where \( \Sigma_R = \Sigma_{R, (\sigma)} + \Sigma_{R, (\omega)} + \Sigma_{R, (\rho)} + \Sigma_{R, (\pi)} \).

- **Effective interactions of DDRHF theory** (8–10 free parameters)

  1. **PKO2**: \( \sigma \)-S and \( \omega \)-V \((m_\sigma, g_\sigma, g_\omega, 3 \text{ density dependent (DD) parameters}); \rho\)-V \((g_\rho, a_\rho)\)

  2. **PKO1, PKO3**: \( \sigma \)-S and \( \omega \)-V \((m_\sigma, g_\sigma, g_\omega, 3 \text{ DD parameters}); \rho\)-V \((g_\rho, a_\rho); \pi\)-PV \((f_\pi, a_\pi)\)

  3. **PKA1**: \( \sigma \)-S and \( \omega \)-V \((m_\sigma, g_\sigma, g_\omega, 3 \text{ DD parameters}); \rho\)-V \((g_\rho, a_\rho); \pi\)-PV \((f_\pi, a_\pi); \rho\)-T \((f_\rho, a_T)\)
**Bogoliubov transformation $\mathcal{W}$:**

$\begin{pmatrix} c_\alpha \\ c_\alpha^\dagger \end{pmatrix} = \mathcal{W} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix} = \begin{pmatrix} \psi_U & \psi_V^* \\ \psi_V & \psi_U^* \end{pmatrix} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix}$.

(12)

**Relativistic Hartree-Fock-Bogoliubov (RHFB) equation:** [H. Kucharek(1991)]

$$\int dr' \begin{pmatrix} h(r, r') - \lambda & \Delta(r, r') \\ \Delta^*(r, r') & -h(r, r') + \lambda \end{pmatrix} \begin{pmatrix} \psi_U(r') \\ \psi_V(r') \end{pmatrix} = E \begin{pmatrix} \psi_U(r) \\ \psi_V(r) \end{pmatrix},$$

(13)

**Single particle Dirac Hamiltonian $h = h^\text{kin} + h^D + h^E$:**

- **Kinetic part:**
  $$h^\text{kin}(r, r') = \left[ \alpha \cdot p + \beta M \right] \delta(r, r'),$$

  (14a)

- **Local Direct part:**
  $$h^D(r, r') = \left[ \Sigma_T(r) \gamma_5 + \Sigma_0(r) + \beta \Sigma_S(r) \right] \delta(r, r'),$$

  (14b)

- **Non-local Exchange part:**
  $$h^E(r, r') = \begin{pmatrix} Y_G(r, r') & Y_F(r, r') \\ X_G(r, r') & X_F(r, r') \end{pmatrix}$$

  (14c)

**Pairing potential and pairing tensor $\kappa$:**

$$\Delta_\alpha(r, r') = -\frac{1}{2} \sum_\beta V^{\text{pp}}_{\alpha\beta}(r, r') \kappa_\beta(r, r'), \quad \kappa_\alpha(r, r') = \psi_{V\alpha}(r)^* \psi_{U\alpha}(r').$$

(15)

**Gogny pairing force:**

$$V(r, r') = \sum_{i=1,2} e^{\left(\frac{(r-r')/\mu_i}{2}\right)^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau)$$
RHFB equations on DWS basis

Dirac Woods-Saxon (DWS) basis:

\[ \psi_U^\kappa = \sum_{p=1}^{N_F} U_p g_p^\kappa + \sum_{d=1}^{N_D} U_d g_d^\kappa, \]

\[ \psi_V^\kappa = \sum_{p=1}^{N_F} V_p g_p^\kappa + \sum_{d=1}^{N_D} V_d g_d^\kappa, \] (16)

Radial RHFB equations → Eigenvalue problem

\[
\begin{pmatrix}
  H - \lambda & \Delta \\
  \Delta & -H + \lambda
\end{pmatrix}
\begin{pmatrix}
  U \\
  V
\end{pmatrix}
= E
\begin{pmatrix}
  U \\
  V
\end{pmatrix},
\] (17)

where \( H \) and \( \Delta \) are \((N_F + N_D) \times (N_F + N_D)\) dimensional matrices, \( U \) and \( V \) are the column vectors with \((N_F + N_D)\) elements.

DWS basis is \textit{ONLY} applied in solving the radial RHFB equations, while Hartree-Fock mean fields are calculated in \textit{Coordinate Space}.

Parameters of DWS basis: \( N_F \) (\( \varepsilon > 0 \)), \( N_D \) (\( \varepsilon < 0 \)), and \( R_{\text{max}} \) (spherical box-size)

\begin{enumerate}
  \item Stable nuclei: \( N_F = 24 \sim 28 \), \( N_D = 12 \), and \( R_{\text{max}} = 20\text{fm} \)
  \item Weakly bound nuclei: \( N_F = 32 \sim 36 \), \( N_D = 12 \), and \( R_{\text{max}} = 24\text{fm} \)
  \item \( N_F \) should be changed with respect to \( R_{\text{max}} \), while \( N_D \) can be fixed.
\end{enumerate}
Results and Discussions

Skin and Halo Phenomena within DDRHFB

Halo and Skin phenomena within RHFB theory

Neutron Skins: Ni and Sn;

W.H. Long (PKU&TUM)

Halos within DDRHFB
Halo and Skin phenomena within RHFB theory

Neutron Skins: Ni and Sn; Neutron halos: Ca, Zr, and Ce
Ordinary and Giant Halos in Cerium isotopes

$\rho_b (\text{fm}^{-3})$

$\nu_{1i} \frac{13}{2}$
$\nu_{2g} \frac{9}{2}$
$\nu_{4s} \frac{1}{2}$
$\nu_{3d} \frac{5}{2}$
$\nu_{3d} \frac{3}{2}$
$\nu_{5s} \frac{1}{2}$

DWS basis: $N_F = 36$, $N_D = 12$, and $R_{\text{max}} = 28 \text{fm}$
Ordinary and Giant Halos in Cerium isotopes

DWS basis: $N_F = 36$, $N_D = 12$, and $R_{\text{max}} = 28 \text{fm}$

$$N_{\text{Halo}} = N_{4s_{1/2}} + N_{3d_{5/2}} + N_{3d_{3/2}}$$
Results and Discussions

Stability of Cerium Halo Structures

Shell Evolution along the chain of Ce

1. Characteristic and consistent isospin dependent behaviors given by PKA1

Proton cores and valence orbits

a. PKA1: proton valence orbits $\pi 1g_{7/2}$, $\pi 2d_{5/2}$ and $\pi 2d_{3/2}$, with the core $Z = 50$
Shell Evolution along the chain of Ce

### Results and Discussions

Stability of Cerium Halo Structures

![Graph showing evolution of ΔE vs N for different models PKA1, PKO1, and DD-ME2](image)

1. Characteristic and consistent isospin dependent behaviors given by PKA1
2. Too strong shell effects at $N = 126$ presented by PKO1 and DD-ME2
3. Unphysically large gap at $Z = 58$ provided by PKO1 and DD-ME2

#### Proton cores and valence orbits

a. PKA1: proton valence orbits $\pi 1g_{7/2}$, $\pi 2d_{5/2}$ and $\pi 2d_{3/2}$, with the core $Z = 50$

b. PKO1 and DD-ME2: *ONLY* proton core $Z = 58$

#### Neutron valence orbits

a. $^{142}\text{Ce} \rightarrow ^{184}\text{Ce}$: $\nu 2f_{7/2}$, $\nu 3p_{3/2}$, $\nu 3p_{1/2}$, $\nu 2f_{5/2}$, $\nu 1h_{9/2}$ and $\nu 1i_{13/2}$

b. $^{186}\text{Ce} \rightarrow ^{198}\text{Ce}$: $\nu 2g_{9/2}$, $\nu 4s_{1/2}$, $\nu 3d_{5/2}$, $\nu 3d_{3/2}$ and $\nu 2g_{7/2}$
Valence Neutron-proton Interaction Strength

1. Neutron orbits with nodes $\nu 2f_{7/2}$, $\nu 3p_{3/2}$, $\nu 3p_{1/2}$, $\nu 2f_{5/2}$ and $\nu 2g_{9/2}$ show stronger couplings with proton state $\pi 2d_{5/2}$ (filled symbols) than with $\pi 1g_{7/2}$ (open symbols).

Figure 1: Total interacting matrix elements $V_{ab}$. The couplings with $\pi 2d_{5/2}$ are denoted with filled symbols, and the ones with $\pi 1g_{7/2}$ are in open symbols.
Valence Neutron-proton Interaction Strength

1. Neutron orbits with nodes $\nu_2f_{7/2}$, $\nu_3p_{3/2}$, $\nu_3p_{1/2}$, $\nu_2f_{5/2}$ and $\nu_2g_{9/2}$ show stronger couplings with proton state $\pi_2d_{5/2}$ (filled symbols) than with $\pi_1g_{7/2}$ (open symbols).

2. Neutron ones without node $\nu_1h_{9/2}$ and $\nu_1i_{13/2}$ present opposite trend.

**Figure 1:** Total interacting matrix elements $V_{ab}$. The couplings with $\pi_2d_{5/2}$ are denoted with filled symbols, and the ones with $\pi_1g_{7/2}$ are in open symbols.
Results and Discussions

Stability of Cerium Halo Structures

Valence Neutron-proton Interaction Strength

-0.2 -0.3 -0.4 -0.5 \( \pi_{2d5/2}, \ldots \)
-0.2 -0.3 -0.4 -0.5 \( \pi_{1g7/2}, \ldots \)
-0.2 -0.3 -0.4 -0.5 \( \nu_{2g9/2}, \ldots \)
-0.2 -0.3 -0.4 -0.5 \( \nu_{1h9/2}, \ldots \)
-0.2 -0.3 -0.4 -0.5 \( \nu_{1i13/2} \)

\( V_{ab} (\text{MeV}) \)

\( N \)

80 88 96 104 112 120 128 136 144

\( \Delta E = E_{\nu_{2g9/2}} - E_{\nu_{1i13/2}} \)

1. Neutron orbits with nodes \( \nu_{2f7/2}, \nu_{3p3/2}, \nu_{3p1/2}, \nu_{2f5/2} \) and \( \nu_{2g9/2} \) show stronger couplings with proton state \( \pi_{2d5/2} \) (filled symbols) than with \( \pi_{1g7/2} \) (open symbols).

2. Neutron ones without node \( \nu_{1h9/2} \) and \( \nu_{1i13/2} \) present opposite trend.

3. Protons in \( \pi_{1g7/2} \) tend to enlarge the shell gap (\( N = 126 \)) between neutron orbits \( \nu_{1i13/2} \) and \( \nu_{2g9/2} \), while \( \pi_{2d5/2} \) presents much less effect.

Figure 1: Total interacting matrix elements \( V_{ab} \). The couplings with \( \pi_{2d5/2} \) are denoted with filled symbols, and the ones with \( \pi_{1g7/2} \) are in open symbols.
1. Appropriate proton configurations, which can only provided by PKA1 with the presence of $\rho$-tensor couplings, guarantee stronger valence neutron-proton ($np$) interactions.

**Table 1:** Two-body interaction matrix elements $V_{ab}$ (MeV) in $^{198}\text{Ce}$.

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<td>$V_{ab}^D$</td>
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<td>$\rho-V$</td>
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<td>$\nu 1_{i}^{13/2}$</td>
<td>-0.257</td>
<td>63.2%</td>
<td>36.8%</td>
<td>9.7%</td>
<td>-4.0%</td>
<td>29.1%</td>
<td>2.1%</td>
<td>-0.386</td>
<td>63.5%</td>
<td>36.5%</td>
<td>8.3%</td>
</tr>
<tr>
<td>$\nu 4s_{1/2}$</td>
<td>-0.097</td>
<td>71.8%</td>
<td>28.2%</td>
<td>7.1%</td>
<td>-4.4%</td>
<td>21.5%</td>
<td>3.9%</td>
<td>-0.052</td>
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<td>-0.159</td>
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Role of Fock terms in the Stability of Halo Structures

1. Appropriate proton configurations, which can only provided by PKA1 with the presence of $\rho$-tensor couplings, guarantee stronger valence neutron-proton ($np$) interactions.

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2. Fock terms enhance the $np$ interactions, among which more than 30% contributions come from the Fock terms in most cases.

3. In the $np$ coupling channel, the Fock terms are totally contributed by isovector mesons, mainly ($\sim 2/3$) by the $\rho$-tensor couplings.
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<td>$V_{ab}$ 63.2% 36.8%</td>
<td>$V_{ab}$ 63.5% 36.5%</td>
</tr>
<tr>
<td>$\nu 4s_{1/2}$</td>
<td>$\rho$-V 9.7% -4.0%</td>
<td>$\rho$-V 9.7% -4.4%</td>
</tr>
<tr>
<td>$\nu 2g_{9/2}$</td>
<td>$\rho$-VT 29.1% 2.1%</td>
<td>$\rho$-VT 25.2% 3.5%</td>
</tr>
<tr>
<td>$\nu 2g_{7/2}$</td>
<td>$\pi$-PV 3.1% 22.9%</td>
<td></td>
</tr>
<tr>
<td>$\nu 3d_{5/2}$</td>
<td>$\nu 3d_{3/2}$</td>
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<td>$\nu 3d_{5/2}$</td>
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</tbody>
</table>

2. Fock terms enhance the $np$ interactions, among which more than 30% contributions come from the Fock terms in most cases.

3. In the $np$ coupling channel, the Fock terms are totally contributed by isovector mesons, mainly ($\sim 2/3$) by the $\rho$-tensor couplings.

4. Within the RHF approach, the Hartree terms contribute stronger bindings than the ones within the RMF approach. [W.H. Long (2006)]

5. Such strong effects due to the Fock terms can not be obtained within the Hartree approach.
SUMMARY

- Relativistic Hartree-Fock-Bogoliubov (RHFB) theory with density dependent meson-nucleon couplings:
  1. Particle-hole channel: the density dependent relativistic Hartree-Fock theory
  2. Particle-particle channel: the finite range pairing force Gogny D1S
  3. RHFB equations are solved by an expansion of the Dirac-Bogoliubov spinors on the Dirac Woods-Saxon (DWS) basis.

- Halo phenomena in Cerium isotopes predicted by RHFB with PKA1
  1. Giant halos (in $^{192,194,196,198}$Ce), as well as the normal ones (in $^{186,188,190}$Ce), are found near the neutron drip line.
  2. Fock terms, mainly $\rho$-tensor couplings, which improve the proton configurations and enhance the neutron-proton interactions, present substantial effects on the stability of halo structures.
  3. The necessity of Fock terms is well demonstrated, since such strong effects can not be obtained within the Hartree approach.

- Things to find out:
  1. Role of Fock terms in stability mechanism of exotic nuclei and superheavy elements
  2. Effects of Fock terms in describing the exotic excitation
  3. Further application in astrophysics such as compact stars
  4. Appropriate pion exchange and $\rho$-tensor couplings