One-Electron Quantum Cyclotron: 
A New Measurement of the Electron Magnetic Moment
and the Fine Structure Constant 

Gerald Gabrielse
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with Shannon Fogwell and Josh Dorr
Earlier contributions: David Hanneke, Brian Odom, Brian D’Urso, Steve Peil, Dafna Enzer, Kamal Abdullah, Ching-hua Tseng, Joseph Tan

2006 DAMOP Thesis Prize Winner

N$F$

0.1 μm
Three Programs to Measure Electron $g$

<table>
<thead>
<tr>
<th>U. Michigan</th>
<th>U. Washington</th>
<th>Harvard</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam of electrons</td>
<td>one electron</td>
<td>one electron</td>
</tr>
<tr>
<td>spins precess with respect to cyclotron motion</td>
<td>4.2 K</td>
<td>quantum jump spectroscopy of quantum levels</td>
</tr>
<tr>
<td>keV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- cylindrical Penning trap
- inhibit spontaneous emission
- measure cavity shifts
- self-excited oscillator

Crane, Rich, … Dehmelt, Van Dyck
References

2006: B. Odom, D. Hanneke, B. D’Urso, G. Gabrielse

2008: D. Hanneke, S. Fogwell and G. Gabrielse
PRL 100, 120801 (2008).

G. Gabrielse, “Determining the Fine Structure Constant”
New Measurement of Electron Magnetic Moment

\[ \bar{\mu} = g \mu_B \frac{\vec{S}}{\hbar} \]

Bohr magneton \( \frac{e\hbar}{2m} \)

\[ g / 2 = 1.00115965218073 \pm 0.00000000000028 \]

2.8 \( \times \) 10\(^{-13} \)

- First improved measurements (2006, 2008) since 1987
- 15 times smaller uncertainty
- 1.7 standard deviation shift
- 2500 times smaller uncertainty than muon g
New Determination of the Fine Structure Constant

\[ \alpha = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\hbar c} \]

- Strength of the electromagnetic interaction
- Important component of our system of fundamental constants
- Increased importance for new mass standard

\[ \alpha^{-1} = 137.035\,999\,084 \pm 0.000\,000\,051 \quad \text{(12) (37) (33)} \]
\[ \pm \quad 3.7 \times 10^{-10} \]

- 20 times more accurate than atom-recoil methods

D. Hanneke, S. Fogwell and G. Gabrielse
Key: Resolving One-Quantum Transitions for One Trapped Electron

One-quantum cyclotron transition spin flip

electron magnetic moment in Bohr magnetons

\[
g \frac{2}{\nu_c} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} = 1 + \frac{\nu_a}{\nu_c},
\]
Twenty Years and 7 PhD Theses?

Takes time to develop new ideas and methods needed to determine $g/2$ to 2.8 parts in $10^{13}$ uncertainty – one PhD at a time

- One-electron quantum cyclotron
- Resolve lowest cyclotron states as well as spin
- Quantum jump spectroscopy of spin and cyclotron motions
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons identify cavity radiation modes
- Trap without nuclear paramagnetism
- One-particle self-excited oscillator
Orbital Magnetic Moment

\[ \vec{\mu} = g \mu_B \frac{\vec{L}}{\hbar} \]

angular momentum

Bohr magneton \( \frac{e\hbar}{2m} \)

magnetic moment

e.g. What is \( g \) for identical charge and mass distributions?

\[ \mu = IA = \frac{e}{2\pi\rho} (\pi\rho^2) = \frac{e\nu\rho}{2} \frac{L}{mv\rho} = \frac{e}{2m} L = \frac{e\hbar}{2m} \frac{L}{\hbar} \]

\[ \Rightarrow \quad g = 1 \]
Spin Magnetic Moment

\[ \vec{\mu} = g \mu_B \frac{\vec{S}}{\hbar} \]

angular momentum

Bohr magneton \( \frac{e\hbar}{2m} \)

\[ g = 1 \quad \text{identical charge and mass distribution} \]

\[ g = 2 \quad \text{spin for simplest Dirac point particle} \]

\[ g = 2.002\ 319\ 304 \ldots \quad \text{simplest Dirac spin, plus QED} \]

(if electron g is different \( \Rightarrow \) electron has substructure)
Why Measure the Electron Magnetic Moment?

1. Electron $g$ - basic property of simplest of elementary particles
2. Determine fine structure constant – from measured $g$ and QED (May be even more important when we change mass standards)
3. Test QED – requires independent $\alpha$
4. Test CPT – compare $g$ for electron and positron $\rightarrow$ best lepton test
5. Look for new physics beyond the standard model
   - Is $g$ given by Dirac + QED? If not $\rightarrow$ electron substructure (new physics)
   - Muon $g$ search for new physics needs $\alpha$ and test of QED
One Electron in a Magnetic Field

$f_c \approx 150 \text{ GHz}$

$n = 1$

$n = 2$

$n = 3$

$n = 4$

$n = 0$

$h \nu_c = 7.2 \text{ kelvin}$

$T \ll 7.2 \text{ K}$

$B \approx 6 \text{ Tesla}$

Need low temperature cyclotron motion

Trap with charges

Gabrielse
Electron Cyclotron Motion Comes Into Thermal Equilibrium

T = 100 mK << 7.2 K \rightarrow \text{ground state always}
Prob = 0.99999…

\begin{align*}
\text{spontaneous emission} & \quad \text{absorb blackbody photons} \\
\text{n = 0} & \quad \text{X} \\
\text{n = 1} & \\
\text{n = 2} & \\
\text{n = 3} & \\
\text{n = 4} & \\
\end{align*}

electron “temperature” – describes its energy distribution on average
Electron in Cyclotron Ground State

QND Measurement of Cyclotron Energy vs. Time

On a short time scale
→ in one Fock state or another

Averaged over hours
→ in a thermal state

average number of blackbody photons in the cavity

First Penning Trap Below 4 K \( \rightarrow \) 70 mK

Need low temperature cyclotron motion
\( T << 7.2 \text{ K} \)
Spin → Two Cyclotron Ladders of Energy Levels

Cyclotron frequency:

\[ \nu_c = \frac{1}{2\pi} \frac{eB}{m} \]

Spin frequency:

\[ \nu_s = \frac{g}{2} \nu_c \]

\[ m_s = -1/2 \quad m_s = 1/2 \]
**Basic Idea of the Fully-Quantum Measurement**

- **Cyclotron frequency:** 
  \[ \nu_c = \frac{1}{2\pi} \frac{eB}{m} \]

- **Spin frequency:** 
  \[ \nu_s = \frac{g}{2} \nu_c \]

- **Measure a ratio of frequencies:**
  \[ \frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} \]

- In free space, the magnetic field cancels out (self-magnetometer)

- \( \approx 10^{-3} \)

- Almost nothing can be measured better than a frequency
Special Relativity Shift the Energy Levels $\delta$

Cyclotron frequency: \[
2\pi v_c = \frac{eB}{m}
\]

<table>
<thead>
<tr>
<th>$n$</th>
<th>$v_c - \delta/2$</th>
<th>$v_c - 3\delta/2$</th>
<th>$v_c - 5\delta/2$</th>
<th>$v_c - 7\delta/2$</th>
<th>$v_c - 9\delta/2$</th>
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</thead>
<tbody>
<tr>
<td>$n = 4$</td>
<td>$v_c - 7\delta/2$</td>
<td>$v_c - 5\delta/2$</td>
<td>$v_c - 3\delta/2$</td>
<td>$v_c - \delta/2$</td>
<td></td>
</tr>
<tr>
<td>$n = 3$</td>
<td>$v_c - 5\delta/2$</td>
<td>$v_c - 3\delta/2$</td>
<td>$v_c - \delta/2$</td>
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<tr>
<td>$n = 2$</td>
<td>$v_c - 3\delta/2$</td>
<td>$v_c - \delta/2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 1$</td>
<td>$v_c - \delta/2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 0$</td>
<td>$v_c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Spin frequency: \[
\nu_s = \frac{g}{2} \nu_c
\]

$m_s = -1/2, m_s = 1/2$

Not a huge relativistic shift, but important at our accuracy
\[
\frac{\delta}{\nu_c} = \frac{h \nu_c}{mc^2} \approx 10^{-9}
\]

Solution: Simply correct for $\delta$ if we fully resolve the levels
(superposition of cyclotron levels would be a big problem)
Cylindrical Penning Trap

\[ V \sim 2z^2 - x^2 - y^2 \]

- Electrostatic quadrupole potential \( \rightarrow \) good near trap center
- Control the radiation field \( \rightarrow \) inhibit spontaneous emission by 200x

One Electron in a Penning Trap

- very small accelerator
- designer atom

\[ v = z^2 - \frac{1}{2}(x^2 - y^2) \]

Electrostatic quadrupole potential

Magnetic field

Electrostatic quadrupole potential

Magnetic field

cool 12 kHz

200 MHz detect

153 GHz need to measure for g/2

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Frequencies Shift

Perfect Electrostatic Quadrupole Trap

\[ \nu_s = \frac{g}{2} \nu_c \]

\[ \nu_c' = \nu_c \]

\[ \nu_z = \nu_c' \]

\[ \nu_m = \nu_z \]

\[ \nu_s = \frac{g}{2} \nu_c \]

Imperfect Trap
- tilted B
- harmonic distortions to V

Problem: \( \frac{g}{2} = \frac{\nu_s}{\nu_c} \) not a measurable eigenfrequency in an imperfect Penning trap

Solution: Brown-Gabrielse invariance theorem

\[ \nu_c = \sqrt{(\nu_c')^2 + (\nu_z)^2 + (\nu_m)^2} \]
An Aside Arising from the Last of These Conferences

Why Is Sideband Mass Spectrometry Possible with Ions in a Penning Trap?

G. Gabrielse*

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA


Measured sideband frequency – function of B tilt and harmonic distortion

\[ \bar{\omega}_+ (\theta, \phi, \varepsilon) + \bar{\omega}_- (\theta, \phi, \varepsilon) \approx \omega_c \]

only an approx.

How bad is this common approximation?

Dimensional analysis:

\[ \Delta \omega_c \sim \omega_c \theta + \omega_c \varepsilon \sim 1\% \]

Using invariance theorem:

\[ \Delta \omega_c \approx \left( \frac{9}{4} \theta^2 - \frac{1}{2} \varepsilon^2 \right) \omega_- \]

much, much smaller

(and a bit of symmetry)
Spectroscopy in an Imperfect Trap

- one electron in a Penning trap
- lowest cyclotron and spin states

\[
\frac{g}{2} = \frac{v_s}{v_c} = \frac{\nu_c + (v_s - \nu_c)}{v_c} = \frac{\nu_c + \nu_a}{v_c}
\]

\[
\frac{g}{2} \approx 1 + \frac{\nu_a - (\nu_z)^2}{2\nu_c} + \frac{3\delta}{2} + \frac{(\nu_z)^2}{2\nu_c}
\]

expansion for \( \nu_c \gg \nu_z \gg \nu_m \gg \delta \)

To deduce \( g \) \( \Rightarrow \) measure only three eigenfrequencies of the imperfect trap
Feedback Cooling of an Oscillator

Electronic Amplifier Feedback: Strutt and Van der Ziel (1942)

Basic Ideas of Noiseless Feedback and Its Limitations: Kittel (1958)

\[
\text{Dissipation: } \Gamma_e = \Gamma(1-g) \quad \text{Fluctuations: } T_e = T(1-g) \\
\text{Fluctuation-Dissipation Invariant: } \frac{\Gamma_e}{T_e} = \text{const}
\]

faster damping rate \rightarrow higher temperature

Applications:
- Milatz, … (1953) -- electrometer
- Dicke, … (1964) -- torsion balance
- Forward, … (1979) -- gravity gradiometer
- Ritter, … (1988) -- laboratory rotor
- Cohadon, … (1999) -- vibration mode of a mirror

Proposal to apply Kittel ideas to ion in an rf trap
- Dehmelt, Nagourney, … (1986) \leftarrow never realized

Proposal to “stochastically” cool antiprotons in trap
- Beverini, … (1988) – stochastic cooling \leftarrow never realized
- Rolston, Gabrielse (1988) – same as feedback cooling (same limitations)

Realization of feedback cooling with a trapped electron (also include noise)
QND Detection of One-Quantum Transitions

\[ \Delta \vec{B} = B_2 z^2 \quad \rightarrow \quad H = \frac{1}{2} m \omega_z^2 z^2 - \mu B_2 z^2 \]

- n=0 cyclotron ground state
- n=1 cyclotron excited state
- n=0 cyclotron ground state

freq = \( E_{cyclotron} = h f_c (n + \frac{1}{2}) \)

n=1

n=0

time
Quantum Non-demolition Measurement

\[ H = H_{\text{cyclotron}} + H_{\text{axial}} + H_{\text{coupling}} \]

\[ [H_{\text{cyclotron}}, H_{\text{coupling}}] = 0 \]

**QND:** Subsequent time evolution of *cyclotron motion* is not altered by additional QND measurements
One-Electron Self-Excited Oscillator

- Axial motion 200 MHz of trapped electron
- Crucial to limit the osc. amplitude

\[ V(t) \]

\[ I^2R \] damping

Amplitude, \( \phi \)
Observe Tiny Shifts of the Frequency of a One-Electron Self-Excited Oscillator

Unmistakable changes in the axial frequency signal one quantum changes in cyclotron excitation and spin

"Single-Particle Self-excited Oscillator"
B. D'Urso, R. Van Handel, B. Odom and G. Gabrielse
Quantum Jump Spectroscopy

- one electron in a Penning trap
- lowest cyclotron and spin states

“In the dark” excitation
→ turn off all detection and cooling drives during excitation

\[ \nu_a = g\nu_c / 2 - \nu_c \]

\[ \nu_c - 3\delta / 2 \]

\[ \nu_c - \delta / 2 \]

\[ \nu_c \]

\[ m_S = -1/2 \quad m_S = 1/2 \]
Inhibited Spontaneous Emission

Application of Cavity QED

\[ \tau = 16 \text{ s} \]
Cavity-Inhibited Spontaneous Emission

Free Space

\[ B = 5.3 \, \text{T} \]

Within Trap Cavity

\[ B = 5.3 \, \text{T} \]

\[ \gamma = \frac{1}{75 \, \text{ms}} \]

\[ \gamma = \frac{1}{16 \, \text{sec}} \]

Inhibition gives the averaging time needed to resolve a one-quantum transition

Inhibited By 210!

Gabrielse, Purcell, Kleppner, Gabrielse and Dehmelt
**Big Challenge: Magnetic Field Stability**

\[ \frac{g}{2} = \frac{\omega_s}{\omega_c} = 1 + \frac{\omega_a}{\omega_c} \]

But: problem when B drifts during the measurement

Magnetic field takes ~ month to stabilize
Self-Shielding Solenoid Helps a Lot

Flux conservation $\Rightarrow$ Field conservation
Reduces field fluctuations by about a factor $> 150$

"Self-shielding Superconducting Solenoid Systems",
Eliminate Nuclear Paramagnetism

Deadly nuclear magnetism of copper and other “friendly” materials

→ Had to build new trap out of silver
→ New vacuum enclosure out of titanium

~ 1 year setback
Measurement Cycle

\[
\frac{g}{2} = \frac{\omega_s}{\omega_c} = 1 + \frac{\omega_a}{\omega_c}
\]

simplified

1. Prepare \( n=0, m=1/2 \) \( \rightarrow \) measure anomaly transition
2. Prepare \( n=0, m=1/2 \) \( \rightarrow \) measure cyclotron transition
3. Measure relative magnetic field

Repeat during magnetically quiet times
Measured Line Shapes for g-value Measurement

It all comes together:

- Low temperature, and high frequency make narrow line shapes
- A highly stable field allows us to map these lines

Precision:

Sub-ppb line splitting (i.e. sub-ppb precision of a g-2 measurement) is now “easy” after years of work
Cavity Shifts of the Cyclotron Frequency

\[ \frac{g}{2} = \frac{\omega_s}{\omega_c} = 1 - \frac{\omega_a}{\omega_c} \]

Within a Trap Cavity

\[ \gamma = \frac{1}{16 \text{ sec}} \]

\( B = 5.3 \text{ T} \)

Spontaneous emission inhibited by 210

cyclotron frequency is shifted by interaction with cavity modes

\( m_s = \frac{1}{2} \)
\( m_s = -\frac{1}{2} \)

n = 3
n = 2
n = 1
n = 0

n = 2
n = 1
n = 0
Cavity modes and Magnetic Moment Error

use synchronization of electrons to get cavity modes

Operating between modes of cylindrical trap where shift from two cavity modes cancels approximately

first measured cavity shift of $g$
2008 Measurement → reduced cavity shift correction uncertainty

• Measuring lifetime and g as a function of B

• Attempting cavity sideband cooling
\[ \left( \frac{g}{2} - 1.00115965218073 \right) \times 10^{12} \]

**x-axis:** Cyclotron frequency / GHz

- **o** without cavity-shift correction
- **•** with cavity-shift correction
New: Careful Study of the Cavity Shifts
Electron as Magnetometer → Lineshapes

\[ \frac{v - \bar{v}_c}{ppb} \quad \frac{v - \bar{v}_a}{ppb} \]
Shifts and Uncertainties $g$ (in ppt = $10^{-12}$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>147.5 GHz</th>
<th>149.2 GHz</th>
<th>150.3 GHz</th>
<th>151.3 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{f}_c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g/2$ raw</td>
<td>-5.24 (0.39)</td>
<td>0.31 (0.17)</td>
<td>2.17 (0.17)</td>
<td>5.70 (0.24)</td>
</tr>
<tr>
<td>Cav. shift</td>
<td>4.36 (0.13)</td>
<td>-0.16 (0.06)</td>
<td>-2.25 (0.07)</td>
<td>-6.02 (0.28)</td>
</tr>
<tr>
<td>Lineshape</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correlated</td>
<td>(0.24)</td>
<td>(0.24)</td>
<td>(0.24)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>uncorrelated</td>
<td>(0.56)</td>
<td>(0.00)</td>
<td>(0.15)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>$g/2$</td>
<td>-0.88 (0.73)</td>
<td>0.15 (0.30)</td>
<td>-0.08 (0.34)</td>
<td>-0.32 (0.53)</td>
</tr>
</tbody>
</table>

- cavity shifts not a problem
- lineshape broadening
New Measurement of Electron Magnetic Moment

\[ \bar{\mu} = g \mu_B \frac{\vec{S}}{\hbar} \]

Bohr magneton \( \frac{e\hbar}{2m} \)

\[ g / 2 = 1.00115965218073 \]

\[ \pm 0.00000000000028 \quad 2.8 \times 10^{-13} \]

- First improved measurements (2006, 2008) since 1987
- 15 times smaller uncertainty
- 1.7 standard deviation shift
- 2500 times smaller uncertainty than muon g
Dirac + QED Relates Measured g and Measured $\alpha$

1. Use measured $g$ and QED to extract fine structure constant
2. Wait for another accurate measurement of $\alpha \rightarrow$ Test QED
Basking in the Reflected Glow of Theorists

\[
\frac{g}{2} = 1 + C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + C_5 \left( \frac{\alpha}{\pi} \right)^5 + \ldots \delta a
\]
“Simple” Analytic Expressions from QED
(where calculations are completed)

\[ A_1^{(4)} = \frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4} \zeta(3) - \frac{\pi^2}{2} \ln(2) \]
\[ = -0.328478965579193 \ldots \]

\[ A_1^{(6)} = \frac{83}{72} \pi^2 \zeta(3) - \frac{215}{24} \zeta(5) - \frac{239}{2160} \pi^4 + \frac{28259}{5184} \]
\[ + \frac{139}{18} \zeta(3) - \frac{298}{9} \pi^2 \ln(2) + \frac{17101}{810} \pi^2 \]
\[ + \frac{100}{3} \left[ \text{Li}_4\left(\frac{1}{2}\right) + \frac{\ln^4(2)}{24} - \frac{\pi^2 \ln^2(2)}{24} \right] \]
\[ = 1.181241456587 \ldots \]

\[ C_8 = A_1^{(8)} = -1.9144 (35) \]

\[ C_{10} = 0.0 (4.6) \]

\( \zeta(s) \) is the Riemann zeta function (Zeta[s] in Mathematica)

polylog: \( \text{Li}_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n} \)
$\frac{g}{2} = 1 + C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + \ldots \delta \alpha$

![Graph showing contributions to g/2 = 1 + α with labels for experimental uncertainty.](image-url)
New Determination of the Fine Structure Constant

\[ \alpha = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\hbar c} \]

- Strength of the electromagnetic interaction
- Important component of our system of fundamental constants
- Increased importance for new mass standard

\[ \alpha^{-1} = 137.035 \ 999 \ 084 \ \pm \ 0.000 \ 000 \ 051 \ 3.7 \times 10^{-10} \]

- 20 times more accurate than atom-recoil methods

D. Hanneke, S. Fogwell and G. Gabrielse
Fine Structure Constant Uncertainties

\[
\alpha^{-1}(H08) = 137.035\,999\,084\,(33)\,(39) \quad [0.24 \text{ ppb}] \quad [0.28 \text{ ppb}],
\]
\[
= 137.035\,999\,084\,(33)\,(12)\,(37) \quad [0.24 \text{ ppb}] \quad [0.09 \text{ ppb}] \quad [0.27 \text{ ppb}],
\]
\[
= 137.035\,999\,084\,(51) \quad [0.37 \text{ ppb}].
\]

(8.14)
Next Most Accurate Way to Determine $\alpha$ (use Cs example)

Combination of measured Rydberg, mass ratios, and atom recoil

$$\alpha \equiv \frac{1}{\frac{4\pi\varepsilon_0}{hc}} e^2$$

$$\alpha^2 = \frac{2R_\infty}{c} \frac{h}{m_e}$$

$$= \frac{2R_\infty}{c} \frac{h}{M_{Cs}} \frac{M_{Cs}}{M_p} \frac{M_p}{m_e}$$

$$\alpha^2 = 4R_\infty c \frac{f_{recoil}}{(f_{D1})^2} \frac{M_{Cs}}{M_{12C}} \frac{M_{12C}}{M_{Cs}} \frac{M_p}{m_e}$$

$$R_\infty \equiv \frac{1}{(4\pi\varepsilon_0)^2} \frac{e^4 m_e c}{2h^3 c^2}$$

- Now this method is >10 times less accurate
- We hope that will improve in the future → test QED

(Rb measurement is similar except get $h/M[Rb]$ a bit differently)
Several Measurements Needed for the Atom-Recoil Determinations of the Fine Structure Constant

Table 8.1. Measurements determining $\alpha(Cs)$.

<table>
<thead>
<tr>
<th>Measurement quantity</th>
<th>$\Delta \alpha/\alpha$ ppb</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_r$ 15</td>
<td>7.7</td>
<td>[22]</td>
</tr>
<tr>
<td>$A_r(e)$ 0.4</td>
<td>0.2</td>
<td>[23, 54]</td>
</tr>
<tr>
<td>$A_r(Cs)$ 0.2</td>
<td>0.1</td>
<td>[18]</td>
</tr>
<tr>
<td>$\omega$ 0.007</td>
<td>0.007</td>
<td>[20]</td>
</tr>
<tr>
<td>$R_{\infty}$ 0.007</td>
<td>0.004</td>
<td>[16, 17, 23]</td>
</tr>
<tr>
<td><strong>Best $\alpha(Cs)$</strong></td>
<td><strong>8.0</strong></td>
<td><strong>[22]</strong></td>
</tr>
</tbody>
</table>

Table 8.2. Measurements determining $\alpha(Rb)$

<table>
<thead>
<tr>
<th>Measurement quantity</th>
<th>$\Delta \alpha/\alpha$ ppb</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_r$ 9.1</td>
<td>4.6</td>
<td>[21]</td>
</tr>
<tr>
<td>$A_r(e)$ 0.4</td>
<td>0.2</td>
<td>[23, 54]</td>
</tr>
<tr>
<td>$A_r(Rb)$ 0.2</td>
<td>0.1</td>
<td>[18]</td>
</tr>
<tr>
<td>$\omega$ 0.4</td>
<td>0.4</td>
<td>[21]</td>
</tr>
<tr>
<td>$R_{\infty}$ 0.007</td>
<td>0.004</td>
<td>[16, 17, 23]</td>
</tr>
<tr>
<td><strong>Best $\alpha(Rb)$</strong></td>
<td><strong>4.6</strong></td>
<td><strong>[21]</strong></td>
</tr>
</tbody>
</table>
Earlier Measurements
(On Much Larger Uncertainty Scale)
Test of QED

Most stringent test of QED: Comparing the measured electron $g$ to the $g$ calculated from QED using an independent $\alpha$

$$\delta g = |g_{\text{experiment}} - g_{\text{theory}}(\alpha)| < 15 \times 10^{-12}$$

- The uncertainty does not come from $g$ and QED, + SM
- All uncertainty comes from $\alpha[\text{Rb}]$ and $\alpha[\text{Cs}]$
- With a better independent $\alpha$ could do a ten times better test
Dear Jerry,

... I love your way of doing experiments, and I am happy to congratulate you for this latest triumph. Thank you for sending the two papers.

Your statement, that QED is tested far more stringently than its inventors could ever have envisioned, is correct. As one of the inventors, I remember that we thought of QED in 1949 as a temporary and jerry-built structure, with mathematical inconsistencies and renormalized infinities swept under the rug. We did not expect it to last more than ten years before some more solidly built theory would replace it. We expected and hoped that some new experiments would reveal discrepancies that would point the way to a better theory. And now, 57 years have gone by and that ramshackle structure still stands. The theorists ... have kept pace with your experiments, pushing their calculations to higher accuracy than we ever imagined. And you still did not find the discrepancy that we hoped for. To me it remains perpetually amazing that Nature dances to the tune that we scribbled so carelessly 57 years ago. And it is amazing that you can measure her dance to one part per trillion and find her still following our beat.

With congratulations and good wishes for more such beautiful experiments, yours ever, Freeman.
In Progress

Better measurement of the electron magnetic moment and the fine structure constant

Better comparison of the positron and electron magnetic moments

Spinoffs

Observe a proton spin flip with one-proton self-excited oscillator (close, see new PRL)

Attempt to make a one-electron qubit (see new PRA)

Goldman talk this afternoon
Close to observing a proton spin flip (we hope).

(Mainz talk by Ulmer is coming.)
New Dilution Refrigerator and Suspension

100 mK trap and suspended electron moves with the 4.2 K superconducting solenoid
New Superconducting Solenoid
Solenoid in Place

Larger
- to allow positron access
- to let electron move with the solenoid

One superconducting lead was broken inside.
⇒ We had to repair
New Silver Trap

Room for positron entry path
Improved cavity mode spectrum (to allow cavity sideband cooling)
Conclusion

Took many years to develop the quantum methods to measure the electron magnetic moment

→ Most precise measurement of the electron magnetic moment
→ Most precise determination of the fine structure constant
→ Most stringent test of QED (higher order and precision)

Soon: Most precise test of CPT invariance with leptons
  More precise electron magnetic moment
  More precise determination of the fine structure constant

Spinoffs: proton magnetic moment
  one-electron qubit